MONTE CARLO ALGORITHM TO STUDY PERFORMANCE PARAMETERS OF SHUTTLE SYSTEMS

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Abstract

Shuttle systems provide alternative solutions in storage technology. Material flow analyses of these systems are extensive due to complex interaction of several shuttles and lift elements. Besides classic cycle time calculation, it also requires advanced analyzing methods.

At the Institute of Logistics Engineering (ITL), Graz University of Technology, an innovative, software-based approach has been developed to investigate the performance of shuttle systems. This approach is based on the Monte Carlo method. The software tool takes into account a variety of different systems and operating parameters to reflect operative behavior of shuttle systems.

1 Introduction

A primary requirement in technical logistics is to realize short throughput times and high flexibility within logistic systems. A specific technical approach to realize these claims is to use autonomous vehicles. A specific application consists in storage technology, where autonomous vehicles are used for storage and retrieval of unit loads in so-called shuttle systems.

Maximum throughput rates and equipment utilization are important parameters for designing automated warehouse equipment [1]. In determining these parameters, it is important to take into account real operating strategies, such as ABC distribution strategies in the warehouse, multiple I/O-points to the pre-storage area, etc. Currently, no ensured analytic calculation approaches to shuttle systems are available in literature to map strategies of this kind. Available information on the performance is based on complex, project-specific individual simulation studies or experience of system suppliers.

Therefore, we (ITL) developed a software-based analysis model based on the Monte Carlo algorithm [6]. The model allows the study of real time behavior of shuttle systems. In this way, the influence of different geometry and operating parameters on the overall system is determined. This makes it possible to examine and compare various planning variants in a very short time.

The following research results are presented in this article:

- Explanation of the evolved analysis model based on the Monte Carlo algorithm
- Explanation of throughput behavior of shuttle systems based on a simple mathematical model
- Comparison of the Monte Carlo Simulation results from the software with the mathematical model
- Explanation of performance behavior of shuttle systems for various system configurations and operating strategies within various analyzing scenarios

2 Description of shuttle system

2.1 System configuration

In technical logistics, shuttle systems are used in static line bay warehouses for storage and retrieval of load units (totes). Individual rack levels are operated by autonomous shuttle elements. A vertical lift element dispatches totes between the storage levels and the input/output positions [4, 5].

The investigated shuttle system consists of the following devices (Figure 1) [5]:

Rack

The rack serves to store load units and comprises certain storage capacity. The rack geometry is defined by length (l_{rack}), height (h_{rack}) and depth. Storage positions are single to multi-fold deep.

Lift

Lift elements dispatch load units vertically along the y-axis. The totes are moved from input point to certain rack levels and are then passed over to a buffer. Furthermore, the totes for retrieval are removed from the buffer and lifted to an output point. Depending on system configuration, one or two lift elements are installed. One or two load-carrying attachments (LCA) are available per lift. Lift elements may operate in single or double cycle mode during storage and retrieval processes.

Shuttle

The shuttle elements are used to transport the load units on the individual rack levels along the x-axis. Each shuttle has a single load-carrying attachment. Each rack level is served by one shuttle. It is assumed that the shuttles cannot leave the rack levels. Shuttle elements may operate in single or double cycle mode during storage and retrieval processes.

Buffer

An input and output buffer with a certain capacity is installed for every rack level. The buffer is used to decouple the operations between shuttle and lift. Nevertheless, waiting times and queuing phenomena may occur in moments of high system utilization.

I/O points
The storage system is connected via one or more I/O points to the storage pre-zone. The positions of the individual I/O points are variable in y direction of the lift.

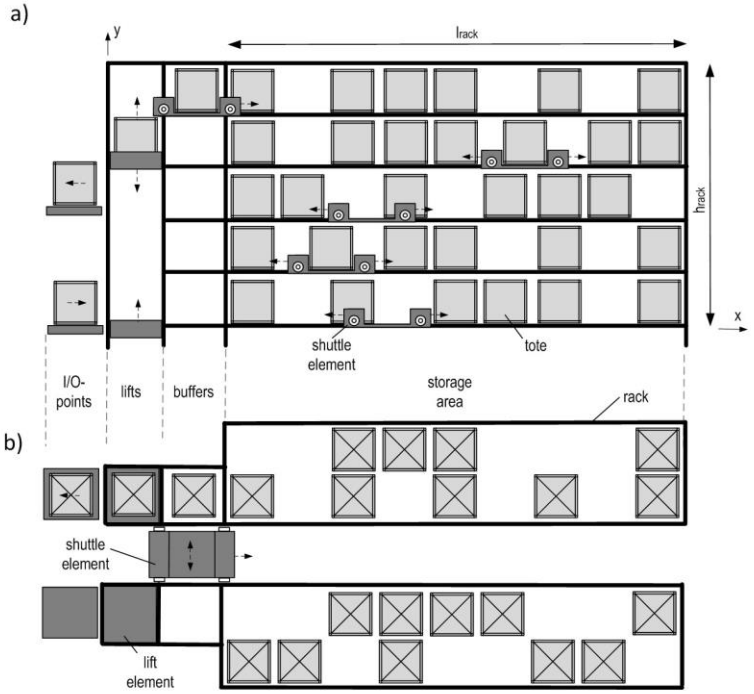

Figure 1: configuration and devices of investigated shuttle system; (a) vertical view, (b) horizontal view

2.2 Storage and retrieval process

The storage and retrieval routines of load units within the system result from interaction of shuttle elements, buffers, and lift. The order of elements assignment in retrieval routines (load units output process) is defined as follows (Figure 2):

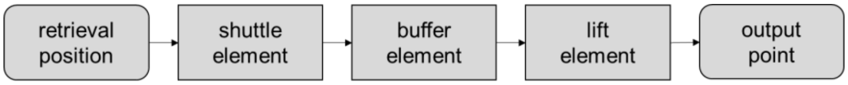

Figure 2: elements assignment in retrieval process

For storage process (load units input process), the elements assignment takes place in reverse direction (Figure 3):

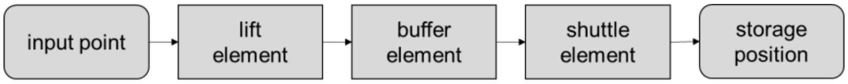

Figure 3: elements assignment in storage process

Shuttle and lift elements perform logic-controlled sequences which are repeated for each storage and retrieval process in the system:

- sequence of input process or output process for shuttle element
- sequence of restoring process for shuttle in multi-deep racks
- sequence of input process or output process for lift element

Exemplary logic-controlled sequences of the shuttle elements are presented in the following.

Sequence of output process for shuttle element

Figure 4a) illustrates the sequence of individual routines for retrieval of load units with shuttle element. Figure 4b) shows the resulting process chain.

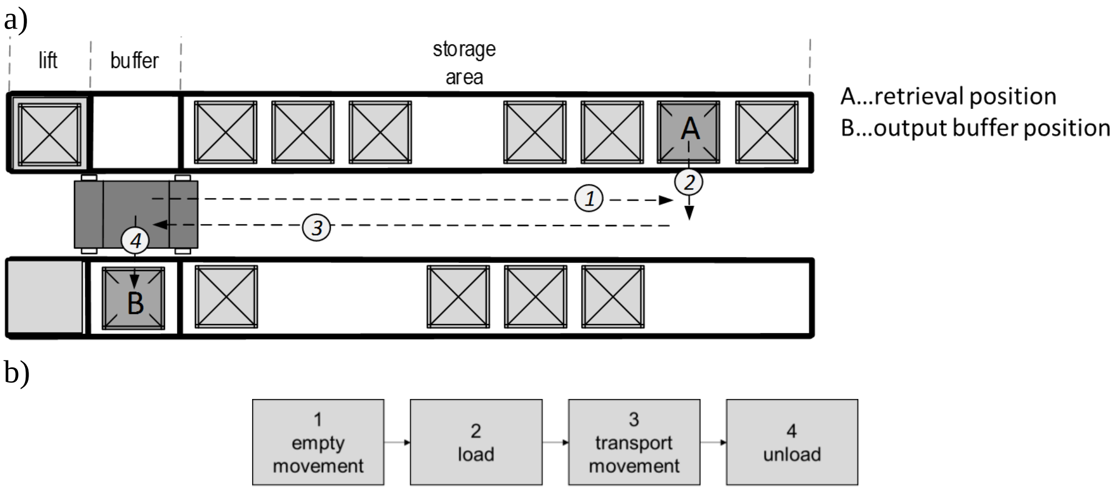

Figure 4: a) sequence of output process for shuttle element, b) resulting process chain

Sequence of input process for shuttle element

Figure 5a) illustrates the sequence of the individual routines for storage of totes for a shuttle element. Figure 5b) shows the resulting process chain.

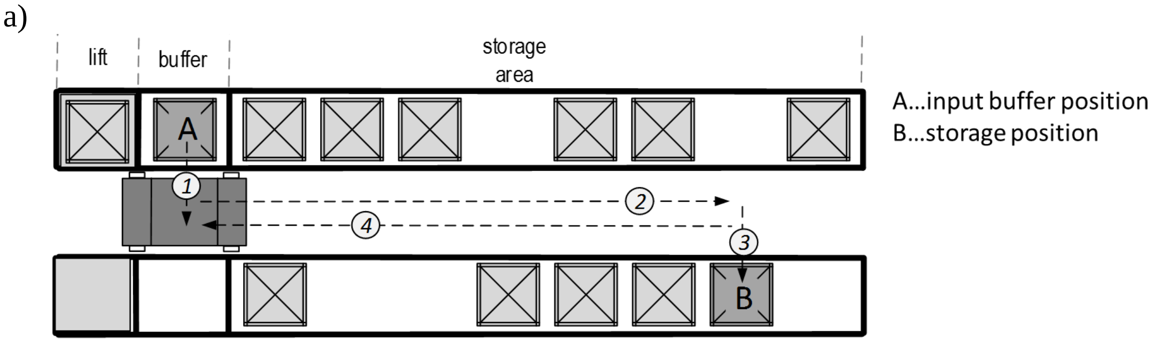

Figure 5: a) sequence of input process for shuttle element

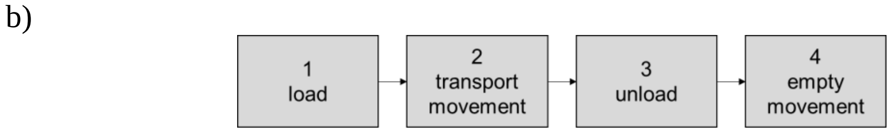

Figure 5: b) resulting process chain for input process

Sequence of restoring process for shuttle in multi-deep racks

For multi-deep racks, it may be necessary to perform a restoring routine to remove totes from a rear rack position (Figure 6):

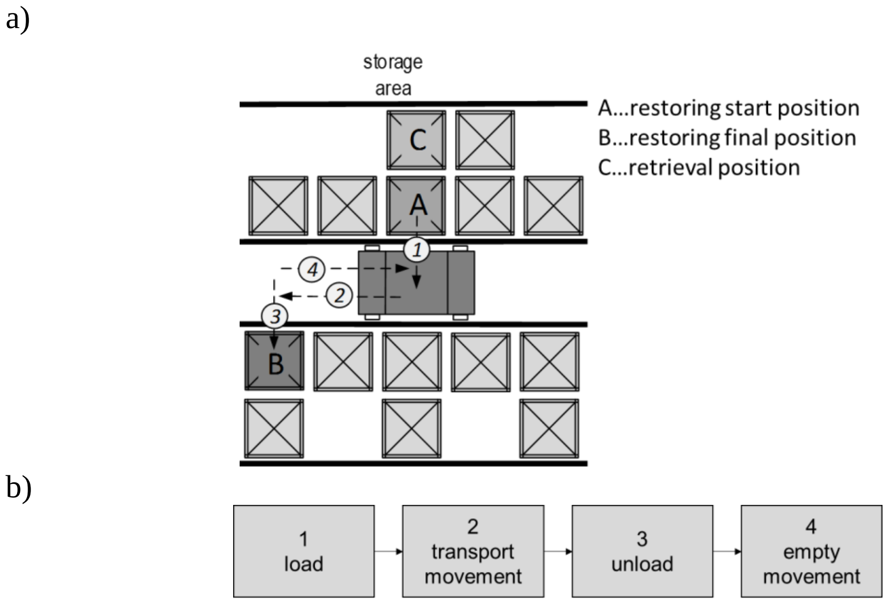

Figure 6: a) sequence of restoring process for shuttle element, b) resulting process chain

3 Monte Carlo approach to study system performance parameters

Shuttle systems demonstrate a complex time behavior during real operations, with a high number of events taking place at the same time. Material handling systems of this type are often too complex for analytical calculation of significant performance parameters.

To analyze the real behavior of such systems mathematically, special methods using stochastic and probability theory have been developed. One of these methods is the Monte Carlo simulation. This approach is based on the numerical calculation of a large number of similar random experiments ('law of large numbers') [2].

The random experiments are conducted on computer calculations using suitable random numbers. From the results of random experiments, the performance parameters of the examined system can be estimated by averaging calculations [3].

This approach is also applicable to the analysis of shuttle systems. Regarding the cycle times ($t_{cycle,i}$) of a large number of samples (n), the expected value of the mean cycle time ($\tau_{cycle} = E(t_{cycle})$) can be determined statistically:

$$\tau_{cycle} = E(t_{cycle}) \approx \frac{1}{n} * \sum_{i=1}^{n} t_{cycle,i} \tag{1}$$

4 Monte Carlo model for analysis of shuttle systems

4.1 Performance calculation of analyzing model

Time specific behavior of the shuttle system depends on logical relations between shuttle elements, buffer elements and lift elements [5]. To calculate the average cycle time, it is relevant to determine individual cycle times of the system (equ. 1). In the following the determination of individual cycle times regarding the outgoing process will be derived.

The single cycle time for one retrieval process is calculated by the sum of the time slices of shuttle element, buffer element and lift element (see Figure 4):

$$t_{cycle,i} = t_{cycle\_shuttle,i} + t_{buffer,i} + t_{cycle\_lift,i} \tag{2}$$

$t_{cycle,i}$ …cycle time for one single retrieval process [s]
$t_{cycle\_shuttle,i}$ …time slice of shuttle movement within the cycle time [s]
$t_{buffer,i}$…time slice of buffer time within the cycle time [s]
$t_{cycle\_lift,i}$…time slice of lift movement within the cycle time [s]

Based on Figure 4 the cycle time of the shuttle ($t_{cycle\_shuttle,i}$) consists of several time slices:

$$t_{cycle\_shuttle,i} = t_{empty\_movement,i} + t_{load,i} + t_{transport,i} + t_{unload,i} \tag{3}$$

$t_{empty\_movement,i}$…time slice for empty movement [s]
$t_{load,i}$…time slice for load movement [s]
$t_{transport,i}$…time slice for transport movement [s]
$t_{unload,i}$…time slice for unload movement [s]

The buffer time ($t_{buffer,i}$) corresponds to the waiting time required by the tote in the buffer waiting to be dispatched by the lift element.
Similar to the shuttle the cycle time of the lift ($t_{cycle\_lift,i}$) consists of the following time slices:

$$t_{cycle\_lift,i} = t_{empty\_movement,i} + t_{load,i} + t_{transport,i} + t_{unload,i} \tag{4}$$

Transport times of shuttle elements and lift elements are determined by standard velocity time characteristics known from literature [5] (Figure 7):

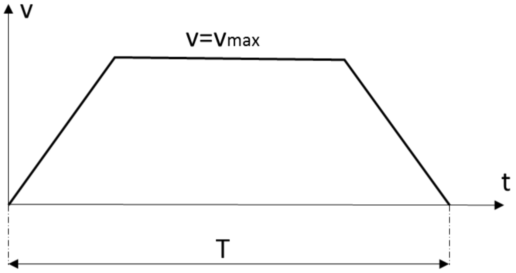

Figure 7: velocity time characteristic of transport elements with variable transport velocity

T...Travel time [s]
t…time [s]
v=v_{max}…maximum velocity of lift or shuttle [m/s]

Calculation of the travel time follows from the following equations:

$$T = \frac{s}{v} + \frac{v^2}{a} \quad for \quad s \geq \frac{v^2}{a} \tag{5}$$

$$T = 2 \cdot \sqrt{\frac{s}{a}} \quad for \quad s < \frac{v^2}{a} \tag{6}$$

with $$a = \frac{2 \cdot a^+ \cdot a^-}{a^+ + a^-} \tag{7}$$

T...Travel time [s]
s...distance between start and target position [m]
v... velocity of lift or shuttle [m/s]
a…acceleration mean for lift or shuttle elements (harmonic mean) [m/s^2]
a^+, a^-...acceleration, deceleration parameters of lift or shuttle [m/s^2]

The utilization of lift elements and shuttle elements (ρ_{lift}, $\rho_{shuttle}$) results from operating time ($t_{operating\_lift}$, $t_{operating\_shuttle}$) and the simulated total time (t_{total}):

$$\rho_{lift} = \frac{t_{operating\_lift}}{t_{total}} \tag{8}$$

$$\rho_{shuttle} = \frac{t_{operating\_shuttle}}{t_{total}} \tag{9}$$

In summary the following significant results about shuttle system performance are calculated within the analyzing model:

- Mean cycle time per storage/retrieval operation
- Average throughput of the shuttle system
- Average utilization rate of shuttles and lifts

4.2 Model configuration parameters

Equipment specification

Each element defined in chapter 2.1 is characterized by several model parameters within the analyzing model influencing system performance:

- parameters of rack (length (l_{rack}), height (h_{rack}), depth of storage, number of rack levels, etc.)
- parameters of shuttle elements (speed and acceleration parameters, load handling and reaction times, etc.)
- parameters of lift elements (speed and acceleration parameters, number of lift elements, number of load carrying attachments, etc.)
- parameters of buffer elements (buffer capacity per rack level, etc.)
- parameters defining the I/O points (number and position of I/O points, etc.)

Processing sequence

The processing sequence defines order and position of individual rack inputs and outputs within the model. For the purpose of the Monte Carlo method, the processing sequence is randomized. The distribution of the random numbers is characterized by distribution functions.

Model parameters for characterizing the processing sequence are:

- rack position allocation for the x/ y/ z position of the shelf (uniform distributed)
- storage/ retrieval positions for x/ y/ z position of the shelf (uniform distributed or ABC-distributed)
- distribution of processing sequence on I/O points (uniform distributed with percentage weighting)

5 Basic model to describe the throughput behavior of shuttle systems

5.1 Model assumptions and calculation

For simple system configurations, the system performance can be described analytically. Regarding a simple model, the throughput of a shuttle system is determined. For this model we consider the system behavior for the output process (unit loads retrieval).

The following assumptions are made for the examined model:

- system works in output mode
- single-depth of storage with variable rack length (l_{rack}) and rack height (h_{rack})
- uniform distributed order positions of outgoing totes in rack (each storage position has the same probability of retrieval)
- one lift element per lane and one shuttle element per rack level for retrieval of load units

In general, the average throughput (μ) for discontinuous working transport elements is calculated from the resulting mean cycle time (τ_{cycle}) [1]:

$$\mu = \frac{1}{\tau_{cycle}} \tag{10}$$

The cycle times ($t_{shuttle}$, t_{lift}) are combinations of individual times for movement of shuttle ($t_{x,shuttle}$) or lift ($t_{y,lift}$), load handling times ($t_{z,shuttle}$ or $t_{z,lift}$) and reaction times ($t_{0,shuttle}$ or $t_{0,lift}$). Each storage position has the same probability of retrieval. The maximum throughput of shuttles ($\mu_{shuttle}$) or lifts (μ_{lift}) is thus calculated from cycle time of shuttle ($t_{shuttle}$) or lift elements (t_{lift}) for half the length or height of the rack.

$$I: \mu_{shuttle} = n_{shuttle} \cdot \frac{3600}{t_{shuttle}(l_{rack}/2)} \quad with \quad t_{shuttle}(l_{rack}/2) = 2 \cdot t_{x,shuttle}(l_{rack}/2) + 2 \cdot t_{z,shuttle} + 2 \cdot t_{0,shuttle} \tag{11}$$

$$II: \mu_{lift} = \frac{3600}{t_{lift}(h_{rack}/2)} \quad with \quad t_{lift}(h_{rack}/2) = 2 \cdot t_{y,lift}(h_{rack}/2) + 2 \cdot t_{z,lift} + 2 \cdot t_{0,lift} \tag{12}$$

l_{rack}…length of rack [m]
h_{rack}…height of rack [m]
$n_{shuttle}$ …number of shuttle elements in total system [#/h] (≙ number of rack levels)
$t_{shuttle}$, t_{lift} …cycle time per shuttle element, cycle time of lift element [s]
$t_{z,shuttle}$, $t_{z,lift}$ …load handling time of shuttle, load handling time of lift
$t_{0,shuttle}$, $t_{0,lift}$ …reaction time of shuttle, reaction time of lift
$\mu_{shuttle}$ …maximum throughput of $n_{shuttle}$ shuttle elements [#/h]
μ_{lift} … maximum throughput of the lift element [#/h]

The equations (equ. 11, equ. 12) show that for constant length and variable height of the rack, the cycle time per shuttle ($t_{shuttle}$) element is constant and the cycle time of the lift (t_{lift}) element is increasing linear. Therefore, the maximum throughput of $x_{shuttle}$ shuttle elements ($\mu_{shuttle}$) is increasing linear and the maximum throughput of the lift element (μ_{lift}) is decreasing non-linear. For constant height and variable length of the rack, the cycle time of the lift element (t_{lift}) is constant and the cycle time per shuttle element ($t_{shuttle}$) is increasing linear. Therefore, the maximum throughput of the lift element (μ_{lift}) is constant and the maximum throughput of $x_{shuttle}$ shuttle elements ($\mu_{shuttle}$) is decreasing non-linear (see chapter 5.2).

Depending on the rack geometry (l_{rack}, h_{rack}), either the shuttles or the lift are the bottle neck elements (degree of utilization of shuttle ($\rho_{shuttle}$) = 100% or degree of utilization of lift (ρ_{lift}) = 100%) limiting the throughput for the shuttle system (μ_{sys}):

$$\mu_{sys} = \begin{cases} \mu_{shuttle}\ if\ \rho_{shuttle} = 100\% \\ \mu_{lift}\ if\ \rho_{lift} = 100\% \end{cases} \tag{13}$$

5.2 Analyzing results

Performance characteristic of the overall system is determined primarily by rack dimensions (l_{rack}, h_{rack}), the number of rack levels and depth of storage.
Regarding constant rack length and varying rack height (corresponds to number of rack levels), the resulting maximum throughput diagram of the system shows three significant sections (Figure 8):

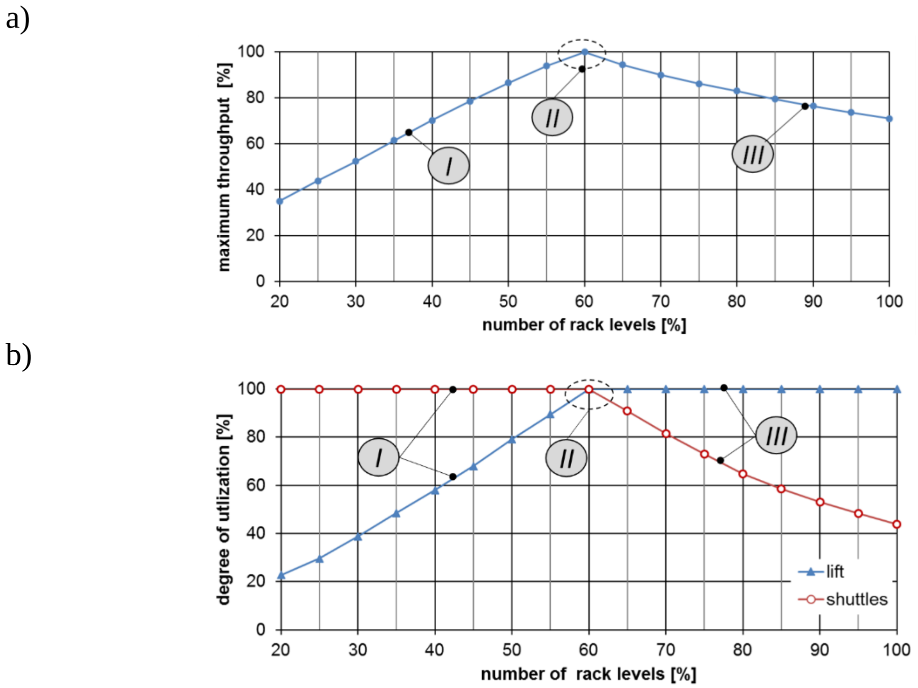

Figure 8: a) maximum throughput of shuttle systems for constant length of the rack and varying number of rack levels, b) degree of utilization of lift and shuttle elements of shuttle systems for constant length of rack and varying number of rack levels

- Section I: in a system with a smaller number of rack levels the maximum throughput is limited by throughput of shuttle elements. Utilization of the shuttle elements ($\rho_{shuttle}$) is therefore 100%, and utilization of the lift element (ρ_{lift}) is below 100%: $\rho_{shuttle} = 100\%$, $\rho_{lift} < 100\%$ (Figure 8b). With an increasing number of rack levels, the system throughput is growing linear.
- Section II: the maximum throughput is achieved at the transition point. Maximum utilization of the lift element changes into maximum utilization of shuttle elements. Utilization of lift and shuttles are identical: $\rho_{lift} \approx \rho_{shuttle} \approx 100\%$
- Section III: with an increasing number of rack levels, the throughput system is limited by throughput of the lift element and is thus decreasing. Utilization of the lift element is therefore 100%, but the utilization of shuttle elements is below 100%: $\rho_{lift} = 100\%$, $\rho_{shuttle} < 100\%$.

Similar throughput characteristics can be seen for constant rack height and variation of rack length (Figure 9):

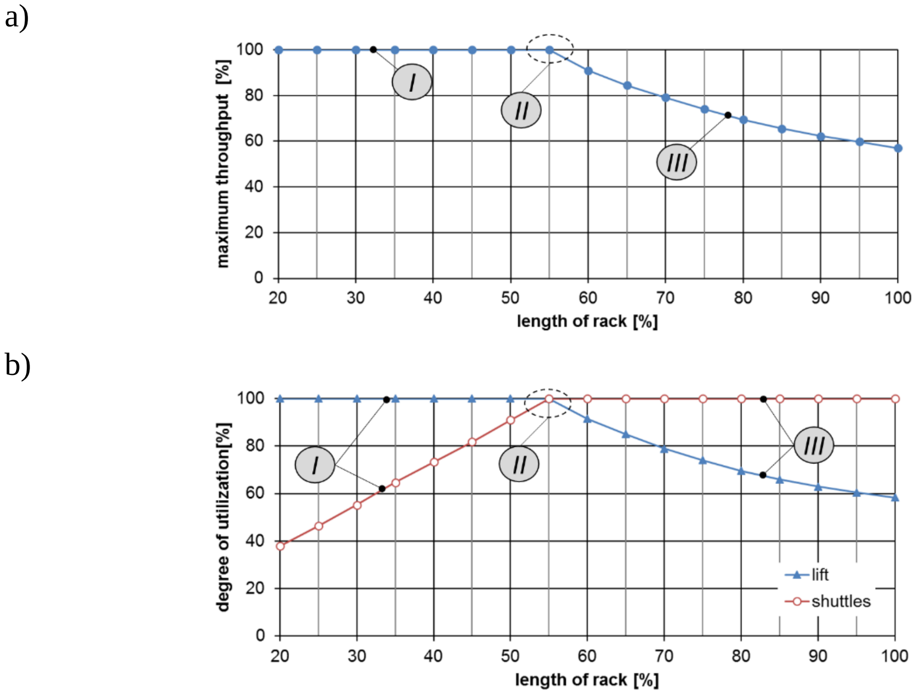

Figure 9: a) maximum throughput of shuttle systems for constant rack height and varying rack length, b) degree of utilization of lift and shuttle elements of shuttle systems for constant rack height and varying rack length

- Section I: in a shorter rack system (relative to rack height), the maximum throughput is limited by the throughput of the lift element and is therefore constant. Utilization of the lift elements is therefore 100%, but the utilization of the shuttles is below 100%: $\rho_{\text{lift}} = 100\%$, $\rho_{\text{shuttle}} < 100\%$ (Figure 9b). With increasing length of the rack, the throughput system remains constant.
- Section II: at the transition point, maximum utilization of the lift element changes into maximum utilization of shuttle elements. Utilization of lift and shuttles are identical: $\rho_{\text{lift}} \approx \rho_{\text{shuttle}} \approx 100\%$
- Section III: with an increasing number of rack levels, the throughput system is limited by throughput of the shuttle elements and is thus decreasing non-linear. Utilization of the shuttle element is therefore 100%, but the utilization of the lift element (ρ_{lift}) is below 100%: $\rho_{\text{shuttle}} = 100\%$. $\rho_{\text{lift}} < 100\%$.

The variation of rack parameters ‘rack length’ and ‘rack height’ (≙ number of rack levels) shows further insights to throughput performance of shuttle systems:

Varying rack height in combination with different constant rack lengths

The following system throughput performance is given by varying rack height in combination with different rack lengths (Figure 10):

- Section I: combinations of low and shorter racks show a higher maximum throughput of the shuttle element than combinations of low and higher racks, since the average transport distances are smaller for the shuttle elements. With increasing rack height (≙ increase of rack levels or shuttle elements), the maximum throughput increases.
- Section II: with short racks - with associated higher throughputs of shuttle elements - and increasing rack height, the lift elements reach the utilization limit faster than with long racks. The maximum throughput therefore drops faster with combinations of shorter and low racks than on longer and low racks.
- Section III: With increasing rack length, the throughput decreases due to the utilization limit of the lift - along with increase in the travel distance of lift. For long racks, throughput curves merge into each other and throughput rate is independent of shelf height - due to the limiting throughput of the lift element.

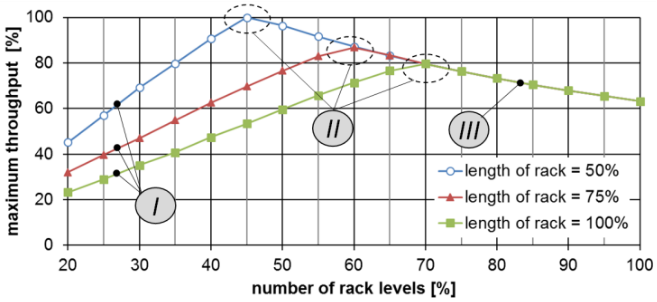

Figure 10: maximum throughput of shuttle systems for three different rack lengths and variable rack height

Varying rack length in combination with different constant rack heights

Similar behavior can be seen in variation of rack length in combination with different constant rack heights (Figure 11):

- Section I: combinations of short and lower racks result in a higher maximum throughput for the lift element than combinations of short and higher racks, since mean transport distances of lift elements are shorter for low racks
- Section II: with increasing rack length, the shuttle elements in lower racks reach utilization limits faster and the maximum throughput decreases earlier than in systems with higher racks (≙ increase of rack levels or shuttle elements)
- Section III: with further increase of rack length, the throughput decreases due to the utilization limit of shuttle elements and increase of mean transport distance of shuttle elements. Combinations of long and higher racks achieve higher throughput than combinations of long and lower racks.

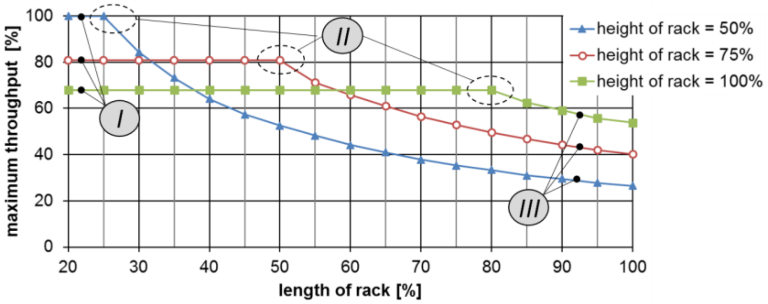

Figure 11: maximum throughput of shuttle systems for three different heights of the rack and variable length of the rack

The results presented are also valid for the input process. The input process runs in reverse direction, but is similar to the output process.

Comparison between the results of the Monte Carlo method with those of the mathematical model shows very good agreement. The differences between the two models are up to 3% maximum. To get result sets of sufficient quality 30.000 input and/or output cycles are simulated per simulation run. The simulation time for one single simulation run takes about 2 seconds.

Complexity of the model increases rapidly with variation of the manipulated variables (system components, operating strategies, etc.), which lead to different configuration options of the overall system. Some interesting parameter studies of the system, which are implemented within the model, are for example:

- Performance behavior for ABC warehouse strategy
- Performance behavior for multi-depth storage with necessary restoring cycles
- Performance behavior of the shuttle system by variation of lift element configuration (number of lift elements and number of LCA)
- Performance behavior by variation of I/O points (number and position)
- Performance behavior for different operation modes of shuttles and lifts (e.g. single cycle mode vs. double cycle mode)

6 Further scenarios for investigation of shuttle systems

To illustrate the analysis capabilities offered within the software tool, further analyzing results are presented within two exemplary scenarios.

Scenario 1

For a shuttle system (according to manufacturer's configuration), with ABC storage distribution the achievable average maximum throughput is analyzed. The ABC rack areas are arranged within rack columns (Figure 12). The ABC distribution is assumed by proportion of consumption and proportion of storage positions (Table 1):

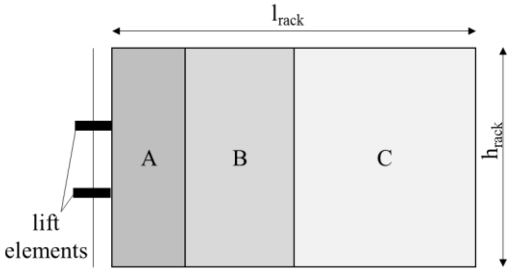

Figure 12: arrangement of ABC warehouse distribution within storage rack

Table 1: assumed ABC storage distribution

ABC storage distribution	Value
proportion of storage positions A	20%
proportion of storage positions B	30%
proportion of storage positions C	50%
proportion of consumption A	70%
proportion of consumption B	20%
proportion of consumption C	10%

By using ABC storage distribution, the utilization of shuttle elements decreases and thus the utilization of the lift increases. The resulting maximum throughput diagram shows three significant sections (Figure 13):

- Section I: For shorter racks the maximum throughput of the system is constant due to throughput limits of the lift elements ($\rho_{lift} = 100\%$, $\rho_{shuttle} < 100\%$).
- Section II: With increasing length of the rack the utilization of the shuttle increases to $\rho_{shuttle} = 100\%$, $\rho_{lift} = 100\%$.
- Section III: As a result, the shuttle elements in racks without ABC warehouse distribution reach utilization limits faster and the maximum throughput decreases earlier than in systems with ABC distribution.

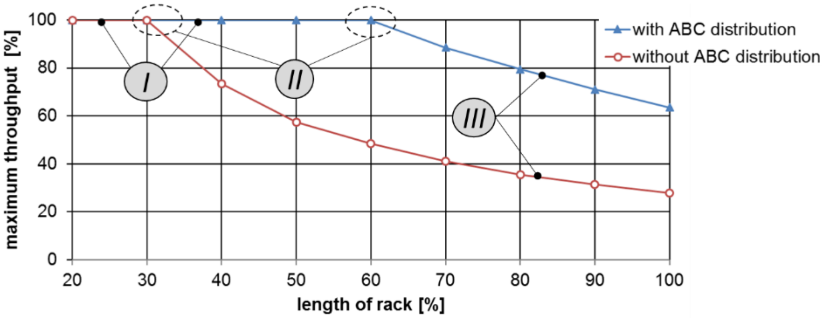

Figure 13: maximum throughput diagram (considering storage strategies with and without ABC warehouse distribution and varying rack length)

Scenario 2

For a shuttle system with multiple storage rack depth (rack depth z = 4) according to manufacturer configurations, the average utilization of shuttle elements is to be determined for retrieval process. The rack occupancy rate (ν_{rack}) and the associated number of necessary rearrangement cycles are defined as varying parameters.

The resulting diagram (Figure 14) shows three significant sections:

- Section I: For lower racks the utilization of the shuttles is $\rho_{shuttle} = 100$ % ($\rho_{lift} <$ 100%).
- Section II: With increasing number of rack levels (≙ increasing height of the rack) the utilization of the lift elements increases to $\rho_{lift} = 100\%$ ($\rho_{shuttle} = 100\%$).
- Section III: With further increase of number of rack levels, the utilization of shuttle elements decreases ($\rho_{shuttle} < 100$ %) - resulting from bottleneck behavior of the lift elements ($\rho_{lift} = 100\%$). As a result, the average utilization of the shuttle elements is higher for higher rack occupancy rates. This is due to the rising number of necessary restoring cycles for retrieval process in multi-deep racks.

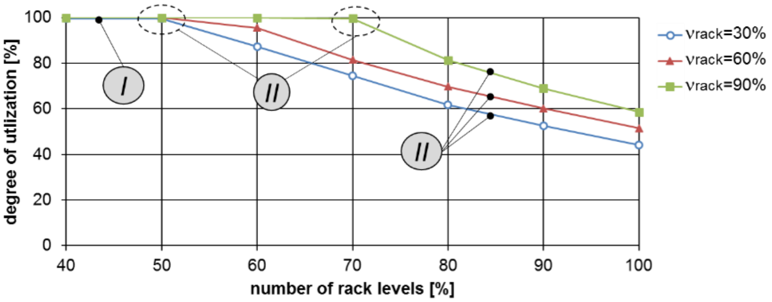

Figure 14: degree of utilization of shuttle elements (considering different rack occupancy rates (ν_{rack}) and varying number of rack levels)

7 Summary and Outlook

This paper presents an appropriate approach to investigate the performance of shuttle systems in early planning stages by using the Monte Carlo algorithm. The approach allows the analysis of real systems with defined system states and fixed system processes where a high degree of variation of the system parameters (geometry, operating strategies, etc.) is possible. It enables a rapid system and analyzing modeling.

We (ITL) are currently in exchange with several system manufacturers to complement the existing model with further parameters and operating strategies of real systems. Therefore, investigations at ITL regarding shuttle systems will be continued. Performance behavior of the systems will be investigated and the operational capability is to be judged in detail. We have also examined other technical systems for storage and retrieval using the mentioned method [3]. We plan to use this approach to investigate other material handling systems (e.g. Carousel System).

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