# A HEURISTIC APPROACH TO STORAGE SYSTEM DESIGN WITH SIMULTANEOUS ASSIGNMENT OF GOODS 

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#### Abstract

In the paper at hand, we analyze the planning of manually operated storage systems consisting of different storage areas. When solving design problems like dimensioning or equipment selection, the assignment of goods to storage areas has to be considered. Based on a general model to describe the geometry and performance calculation of a warehouse, we present a heuristic approach to assign goods to storage areas while simultaneously determining the design of the storage areas in a way that generates the lowest running costs. Our approach aims to support planners during the rough planning stage by providing a quick overview of different solution alternatives. To evaluate our approach, we apply it to an example from industrial practice.


## 1 Introduction

Within supply chains, there are many reasons to store goods. As buffers, storage can decouple production steps and thus stabilize the production process. Storage areas for finished products increase the delivery capability. For order picking, storage areas are needed to house goods to be picked [1]. Due to the variety of applications and goods to be stored, there are numerous different system alternatives available. Basically, there are two groups: Automated systems are advantageous when high performance is needed and compensate for the high investment. Manually operated systems, on the other hand, are still an important part of today's supply chains whenever flexibility is needed: low-level picker-to-part systems for example, in which a picker travels along aisles, cover $80 \%$ of all order picking systems in Western Europe and are estimated to be a cost-intensive aspect of warehouses, attributed to $55 \%$ of the operating costs [2]. We therefore focus on manual systems in the paper at hand, as suitably planning these systems is of great importance in order to avoid additional costs. These costs are incurred, for example, if the service level of the system is insufficient or an inaccurately planned warehouse has to be re-planned.

The task of planning warehouses comprises design and operation problems [3]. For each storage area of the warehouse, the design problems are sizing and dimensioning, storage area layout, equipment selection, and operation strategy; additionally, operation problems have to be solved by deciding about storage and order picking. However, there are strong interactions between the decisions that are to be made when solving the design and operation problems [4]. For example, the storage equipment selection of a storage area requires information about the goods that are assigned to it.

In this paper, we address the problem to assign goods to storage areas while simultaneously solving the design problems previously mentioned. The approach we present follows a two-step pattern of generating different solution alternatives and subsequently evaluating them [5]. Each solution alternative corresponds to a warehouse consisting of storage areas with different design. To evaluate the potential of each solution alternative, we use running costs as a figure based on which solution alternatives can be ordered on a scale. Furthermore, we focus on the rough planning stage of manual systems in which the decisions of the design problems have to be made.

The paper starts with an overview of the state of the art of planning manual systems. Based on the scope of our research, models to evaluate geometry, performance and running costs of a warehouse are described. We then consider the problem of planning manual systems as an optimization problem and analyze its complexity. With regard to the solvability, we present a heuristic solution technique. Applying the solution technique to an existing warehouse then allows us to discuss its applicability.

## 2 Literature

In the literature, numerous models have been presented to support warehouse planning by addressing one or more of the design and operation problems previously mentioned. We give an overview focusing on the problems in our scope: Sizing and dimensioning, storage area layout, equipment selection, operation strategy, and assignment of goods to storage areas.

As the size of each storage area depends on the space requirement of the assigned goods, the problem of sizing and dimensioning the storage areas comprises the translation of "capacity into floor space in order to assess construction and operating costs [4]." The models presented in [6] and [7] aim to minimize the costs for construction and handling of a warehouse with a single storage area. The cost functions for handling model typical cycles of a unit load process. However, the variables considered do not cover storage area layout, equipment and operation strategy in detail.

The problems of storage area layout, equipment selection, and operation strategy affect the costs due to the space requirement, capacity of the storage equipment and the required number of storage/retrieval vehicles (S/RV). A model to determine warehouse bay configurations to minimize costs of construction and handling is discussed in [8]. A detailed analytical model to describe the storage area layout for different types of storage equipment is given in [9]. The number of S/RV required to achieve a given throughput
also depends on storage area layout and equipment selection, since longer distances to cover in a cycle lower the performance of a S/RV. The length of picking routes in lowlevel picker-to-part systems has been studied in [10] and [11], assuming randomly distributed pick locations. To cover more routing strategies and general pick location distribution, new models have been derived from their work, e.g. in [12; 13; 14; 15]. In high-level picker-to-part systems, pickers are additionally able to move up and down, which allows for different routing strategies in an aisle [16]. In unit load systems, a cycle only consists of up to two stops. Travel time calculation is considerably easier, but depends on whether the storage/retrieval vehicles (S/RV) can move in two dimensions simultaneously. Models considering both cases have been developed [9; 17]. However, there is no analytical model covering all variations of manual systems (low-level/highlevel order picking, unit load systems) at the same time.

For the assignment of goods to storage areas we assume that the decision is not straightforward. This would be the case if e.g. one storage area is assigned to one customer and therefore has to contain his assortment exclusively [3]. Research about this problem focusses on the forward-reserve problem that treats the assignment of load units (LU) to a picking area and to a connected reserve area, respectively. Since our scope does not comprise this aspect, research from this field does not contribute to the problem we address. In the literature that addresses the other design and operation problems, assignment is assumed to be fixed in advance, e.g. in [9], since a certain capacity of the storage area is required.

Unlike literature on one or more of the design and operation problems of warehouse planning, general design methodologies are seldom described. A hierarchical approach is proposed in [18]. Depending on the systems that are to be considered, different models to solve design and operation problems can be used on the hierarchical levels. [19] introduces an axiomatic warehouse design theory, which provides a framework for warehouse design models. An abstract data model is discussed in [20], providing a basis for arbitrary design and operation models

With regard to that situation, a lack of models is stated that include the decision of assigning goods to storage areas while designing them simultaneously. Such a model could also be used as part of general design methodologies like in [18], [19] and [20].

## 3 Problem Statement

In the rough planning state, planners often make decisions based on their experience [21], but exclude potentially good solution alternatives in order to keep the effort manageable. On the contrary, many approaches from research refer to only a particular system and exclude the assignment of goods to different storage areas. We address this gap between industrial practice and research, as a fast and comprehensive computer-aided analysis of a large number of solution alternatives provides the planner with broad knowledge about his/her options. Thus, the problem we address in this paper can be stated as follows:

- Is it possible to find a general formulation as an optimization problem to solve the problems of sizing and dimensioning, storage area layout, equipment selection, operation strategy, and assignment of goods to storage areas simultaneously?
- Is the optimization problem solvable in acceptable time and for expected problem sizes from industrial practice, and which solution technique is appropriate for finding a solution?

The choice of the solution alternative with the highest potential requires a figure that is measureable on a metric scale. For a warehouse consisting of $m$ storage areas and containing n goods, running costs can be used as this figure, since they include investment in construction, storage equipment, and S/RV as well as maintenance and employees. The lower the costs, the better a solution alternative is. Our problem of planning manual systems can thus be transferred into a problem of minimizing costs:

$$
\begin{equation*}
\operatorname{Min} \mathrm{C}(X, \mathrm{D}) \tag{3-1}
\end{equation*}
$$

with the constraints:

$$
\begin{gather*}
\sum_{i=1}^{m} x_{i j}=1 \forall j=\{1 \ldots n\}  \tag{3-2}\\
\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{ij}} \geq 1 \forall \mathrm{i}=\{1 \ldots \mathrm{~m}\}  \tag{3-3}\\
\mathrm{x}_{\mathrm{ij}} \in\{0 ; 1\}  \tag{3-4}\\
\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{x}_{\mathrm{ij}} * \mathrm{f}_{\mathrm{ij}}\left(\mathrm{x}_{\mathrm{ij}}, \underline{d_{i}}\right)=1 \forall \mathrm{j}=\{1 \ldots \mathrm{n}\} \tag{3-5}
\end{gather*}
$$

where
$\mathrm{C}(\mathrm{X}, \mathrm{D})$ running costs of the warehouse depending on storage area design and article assignment.
$\mathrm{X} \quad$ assignment matrix with $\mathrm{m} * \mathrm{n}$ elements $\mathrm{x}_{\mathrm{ij}}$ defining the assignment of goods to storage areas.
$\mathrm{x}_{\mathrm{ij}} \quad$ binary variable for the assignment of article j to storage area $\mathrm{i} ; \mathrm{x}_{\mathrm{ij}}=1$ if article $j$ is assigned to storage area $i$ and 0 otherwise.
D design matrix defining the design all storage areas of the warehouse by m vectors $\underline{d}_{i}$.
$\underline{d}_{i} \quad$ design vector defining the design of storage area $i$; the design vector contains all variables regarding design, strategies, and dimensions.
$\mathrm{f}_{\mathrm{ij}}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{d}}_{\mathrm{i}}\right)$ binary variable for the feasibility to store article j to storage area i with design $\underline{\mathrm{d}}_{\mathrm{i}} ; \mathrm{f}_{\mathrm{ij}}=1$ if article j can be stored in storage area j and 0 otherwise.
(3-2) to (3-5) constrain the assignment of goods to storage areas: As goods can only be assigned in full, (3-4) demands $\mathrm{x}_{\mathrm{ij}}$ to assume only binary values. Additionally, every article has to be assigned to one storage area (3-2), while "empty" storage areas without
at least one assigned article are not considered (3-3). With (3-5), we ensure that every article can be stored in the assigned storage area, using a binary variable to describe the feasibility of the assignment.

## 4 The General Warehouse Model

To solve the problem defined by (3-1) to (3-5), the cost function $\mathrm{C}(\mathrm{X}, \mathrm{D})$ has to be specified further. We assume that all storage areas of a warehouse are independent from each other regarding their geometry and performance. Orders are thus divided into partial orders for each storage area, containing only goods stored there. We further define that a warehouse is a feasible solution alternative if each storage area provides enough capacity for all assigned goods and enough S/RV for all orders containing the assigned goods. Running costs of a feasible warehouse are thus given by adding the costs of each storage area:

$$
\begin{equation*}
\mathrm{C}(X, D)=\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{C}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right) \tag{4-1}
\end{equation*}
$$

As the costs of the warehouse/storage are used to distinguish solution alternatives, only those aspects are taken into account that depend on design and assigned goods. For each storage area, we consider the following terms:

- Area costs per period are linearly dependent on the area $A$, where $c_{A}$ contains the price per square meter and period:

$$
\mathrm{C}_{\mathrm{i}, \mathrm{~A}}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)=\mathrm{c}_{A} * \mathrm{~A}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)
$$

- Construction costs per period are linearly dependent on both the area A and the volume V of the storage area. $\mathrm{c}_{\mathrm{B}, 1}$ contains the price per square meter and period of the floor slab. $\mathrm{c}_{\mathrm{B}, 2}$ contains the price for one cubic meter of building per period:

$$
C_{i, B}\left(x_{i j}, d_{i}\right)=c_{B, 1} * A\left(x_{i j}, d_{i}\right)+c_{B, 2} * V\left(x_{i j}, d_{i}\right)
$$

- Storage equipment costs are linearly dependent on the capacity CA [9], where $\mathrm{cs}_{\mathrm{s}}$ contains the price per storage space for one load unit (LU) per period:

$$
\mathrm{C}_{\mathrm{i}, \mathrm{~S}}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)=\mathrm{c}_{\mathrm{S}} * \mathrm{CA}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)
$$

- Costs for S/RV and employees are linearly dependent on the number N of $\mathrm{S} / \mathrm{RV}$ needed to perform all orders assigned to the stored goods. cv contains the costs per S/RV of a certain type and employee per period:

$$
\mathrm{C}_{\mathrm{i}, \mathrm{~V}}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)=\mathrm{c}_{\mathrm{V}} * \mathrm{~N}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)
$$

- Additional costs $\mathrm{C}_{\mathrm{C}}$ contain any cost independent from area, volume, capacity and number of vehicles.

Storage area costs from (4-1) can be written as follows using the cost terms given above:

$$
\begin{gather*}
\mathrm{C}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)=\left(\mathrm{c}_{\mathrm{A}} * \mathrm{c}_{\mathrm{B}, 1}\right) * \mathrm{~A}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)+\mathrm{c}_{\mathrm{B}, 2} * \mathrm{~V}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)+\mathrm{c}_{\mathrm{S}} * \mathrm{CA}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)  \tag{4-2}\\
+\mathrm{c}_{\mathrm{V}} * \mathrm{~N}\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{~d}_{\mathrm{i}}\right)+\mathrm{C}_{\mathrm{C}}
\end{gather*}
$$

With (4-2), (4-1) shows a linear dependency of warehouse costs from area, volume, capacity and number of S/RV of each storage area. To calculate these variables, fitting partial models for geometry and handling are included in the general warehouse model.

The partial model of geometry has been designed to describe the geometry of a storage area based on a given article assignment and design. As the scope of our research comprises manual systems, we include storage equipment typically used in such systems, e.g. ground storage, pallet rack and shelving systems.

We chose the findings in [9] as a basic approach for our partial model of geometry. It is closely related to the physical structure of the storage equipment, using storage spaces (SS), shelf bays (SB), aisle units (AU) and storage areas (SA) for a modular set of components from which the storage area is built stepwise. When adapting the model, we were able to show that it can be used for all types of storage equipment in the scope of our project by defining two types of storage spaces, aisle units and storage area layouts (see Figure 2) [22].


Figure 2: Modular Library of the Partial Model of Geometry [22]

The partial model provides area, volume and capacity as needed for (4-2) as

$$
\begin{align*}
& \mathrm{A}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)=\mathrm{f}\left(\mathrm{l}_{\mathrm{SS}}, \mathrm{w}_{\mathrm{SS}}, \mathrm{~h}_{\mathrm{SS}}, \mathrm{l}_{\mathrm{AU}}, \mathrm{w}_{\mathrm{SA}}, \mathrm{~h}_{\mathrm{SB}}\right)  \tag{4-3}\\
& \mathrm{V}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)=\mathrm{g}\left(\mathrm{l}_{\mathrm{SS}}, \mathrm{w}_{\mathrm{SS}}, \mathrm{~h}_{\mathrm{SS}}, \mathrm{l}_{\mathrm{AU}}, \mathrm{w}_{\mathrm{SA}}, \mathrm{~h}_{\mathrm{SB}}\right) \tag{4-4}
\end{align*}
$$

$$
\begin{equation*}
\mathrm{CA}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)=\mathrm{h}\left(\mathrm{l}_{\mathrm{SS}}, \mathrm{w}_{\mathrm{SS}}, \mathrm{~h}_{\mathrm{SS}}, \mathrm{l}_{\mathrm{AU}}, \mathrm{w}_{\mathrm{SA}}, \mathrm{~h}_{\mathrm{SB}}\right) \tag{4-5}
\end{equation*}
$$

where
$1_{\text {SS }}, \mathrm{w}_{\mathrm{SS}}$, h hss length, width and height of a storage space in load units, multiplying to determine the capacity of a storage space CAss; for example, in a block storage with load units stacked four levels high (hss $=4$ ) with one block four stacks long $\left(l_{s s}=4\right)$ and three stacks wide ( $\mathrm{Wss}=3$ ), storage space capacity is 48.
$1_{\mathrm{AU}} \quad$ length of one aisle unit in storage spaces.
wSA width of the storage area in aisle units, equaling the number of parallel aisles.
$h_{S B} \quad$ height of one shelf bay in storage spaces.
Each of the problems we address in our paper affects the shape of the functions $f, g$ and $h$ in (4-3) to (4-5). Switching the orientation of the aisles from perpendicular to parallel for example doubles the number of aisles units, but halves their length to keep the storage area capacity constant. Additionally, it causes the front aisle area to be dependent on the length of the aisle units ( $l_{\mathrm{AU}}$ ) rather than the width of the storage area ( $\mathrm{w}_{\mathrm{SA}}$ ).

The partial model for performance evaluation uses particular properties of the geometry, such as the length and height of the storage area, as input. The tasks of this partial model are to describe all handling processes within a storage area and to derive cycle time and the number of $\mathrm{S} / \mathrm{RV}$ needed to perform all orders. In general, there are three approaches to calculate the handling times: Estimation, analytic formulas, and simulation. For our partial model we decided to use the approach of analytical formulas, since it allows us to include analytical formulas in the objective function. If, in contrast, cycle times are calculated based on a simulation, results from intermitting simulation experiments would be needed to calculate the number of vehicles and employees N .

Our partial model for performance evaluation is based on three analytical models for cycle time calculation, as there are no analytical models known to cover low-level order picking, high-level order picking and unit load systems. We therefore chose the following models, adapting them to our partial model for geometry:

- Low-level order picking: Formulas for five routing strategies and pick locations both equally distributed as well as ordered by decreasing picking frequency [15].
- High-level order picking: Formulas for three routing strategies in a rack system with equally distributed pick locations [16].
- Unit load systems: Formulas for single or dual cycles typical for order picking [9].

The general approach behind the three models is similar: Based on the most probable storage spaces, the S/RV has to move to during a mean cycle, the distance as well as the required time are calculated. The performance of one $\mathrm{S} / \mathrm{RV}$ is inversely proportional to the cycle time and defined as

$$
\begin{equation*}
\mathrm{P}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)=\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~T}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)} \tag{4-6}
\end{equation*}
$$

where
$\mathrm{P}\left(\mathrm{x}_{\mathrm{i}}, \underline{\mathrm{d}}_{\mathrm{i}}\right)$ performance of one $\mathrm{S} / \mathrm{RV}$ under the given geometry, assortment and order data.
$\mathrm{N}_{\mathrm{P}} \quad$ number of picks/LU moved per cycle.
$\mathrm{T}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{d}}_{\mathrm{i}}\right)$ mean time of one cycle depending on assignment of goods and storage area design.

The number of required $\mathrm{S} / \mathrm{RV}$ and employees is defined as the ratio of the target performance of the storage area and the performance of one $\mathrm{S} / \mathrm{RV}$ :

$$
\begin{equation*}
\mathrm{N}=\frac{\mathrm{P}_{\mathrm{T}}\left(\mathrm{x}_{\mathrm{ij}}\right)}{\mathrm{P}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{~d}_{\mathrm{i}}}\right)} \tag{4-7}
\end{equation*}
$$

where
$\mathrm{N} \quad$ number of $\mathrm{S} / \mathrm{RV}$ and employees needed to perform all order picks.
$\mathrm{P}_{\mathrm{T}}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{d}}_{\mathrm{i}}\right)$ target performance of the storage area in which the $\mathrm{S} / \mathrm{RV}$ are operating, depending on the target performance of the assigned goods.
As we showed for the calculation of $\mathrm{A}, \mathrm{V}$ and CA in (4-3) to (4-5), the shape of the functions $\mathrm{P}_{\mathrm{T}}\left(\mathrm{x}_{\mathrm{ij}}\right)$ and $\mathrm{P}\left(\mathrm{x}_{\mathrm{ij}}, \underline{\mathrm{d}}_{\mathrm{i}}\right)$ also depends on decisions made to solve the problems of planning. Switching the storage equipment of a unit load storage area from block storage with reach trucks to pallet racks allows for high shelf stackers to be considered as alternative $\mathrm{S} / \mathrm{RV}$. As high shelf stackers are able to move in two dimensions simultaneously, the mean time of one cycle is calculated differently [9]. Another example is the effect of the movement strategy on the cycle time in low-level order picking. For the traversal strategy without the possibility to skip aisles in which no good has to be picked, $T\left(\mathrm{x}_{\mathrm{ij}}, \mathrm{d}_{\mathrm{i}}\right)$ is dependent on the number and length of aisles [15]:

$$
\begin{equation*}
\mathrm{T}_{\text {Trav,no skip }}=\left(\text { const }+\mathrm{l}_{\mathrm{AU}}\right) * \mathrm{w}_{\mathrm{SA}} \tag{4-3}
\end{equation*}
$$

If instead the traversal strategy is applied with the possibility to skip unnecessary aisles, an additional term is added, modelling the probability of how many aisles can be skipped [15]:

$$
\begin{equation*}
\mathrm{T}_{\text {Trav,skip }}=\left(\text { const }+\mathrm{l}_{\mathrm{AU}}\right) * \mathrm{w}_{\mathrm{SA}} *\left[1-\frac{\binom{\mathrm{M}-\mathrm{M}_{\mathrm{G}}}{\mathrm{n}}}{\binom{\mathrm{M}}{\mathrm{n}}}\right] \tag{4-4}
\end{equation*}
$$

where
n number of goods to be picked in one cycle.
M number of goods in the storage area depending on assignment matrix X .
$\mathrm{M}_{\mathrm{G}} \quad$ number of goods in one aisle depending on assignment matrix X and aisle unit capacity.

## 5 Complexity of the Optimization Problem

Based on the general warehouse model, we are able to answer the first part of the problem statement: planning manual systems can be formulated as an optimization problem to minimize costs for area, construction, storage equipment, S/RV, employees and additional constant aspects. As we determined the wide applicability of manual systems is to be preserved, there is no manageable formulation of the objective function. Area, volume, capacity and cycle time are calculated in different ways depending on the storage area design $\underline{\mathrm{d}}_{\mathrm{i}}$. Introducing new binary variables as factors to activate the alternative terms (e.g. (4-3) and (4-4)) depending on the chosen design (e.g. routing strategy) is possible, but makes the objective function hardly manageable.

To show the complexity of the problem we address, we simplify it by assuming a given assignment of goods to storage areas and by considering only one of the m storage areas. We predefine all design and operation variables that define the shape of the terms to be used in that particular case. In this simplified case, the objective function only depends on the dimensions of storage spaces and storage area, but is generally non-linear (e.g. with (4-4) and (4-6) inserted into (4-7)) and contains integer variables (e.g. the number of aisles). We can thus sum up that we have to solve multiple mixed-integer nonlinear programs (MINLP), since the predefined assignment of goods and design/operation variables have to be varied to cover the whole solution set and to provide the optimum.

The number of MINLP to be solved depends on the number of possible assignments of goods to storage areas, and on the number of possible combinations of design and operation variables. As not all combinations of goods and/or values of the design and operation variables are feasible for each given assortment of goods, we can only give an upper boundary. Assume all $n$ goods can be stored together or separately, resulting in a number of storage areas $m$ between 1 and $n$. To calculate the exact number, we start with all partitions of $n$, which is the number of distinct sums of natural numbers with the result n . The number of partitions is given by the partition function $\mathrm{P}(\mathrm{n})$, which is exponentially growing for large $n$ [23]. For example, there are seven partitions of $5(5,4+1,3+2, \ldots)$. The number of summands of a partition determines the number of storage areas. We then have to apply an urn problem to determine which goods are assigned to which of the storage areas. For $4+1$, there are five possibilities which of 5 goods is stored separately, and which 4 of 5 goods are stored together. In total, there are 67 different assignments for 5 goods.

For each storage area of each possible assignment, multiple MINLP have to be solved, due to varying the formerly predefined design and operation variables. For one storage area with order picking, for example, there are more than 1,000 sets of values for the design and operation variables, which corresponds to an equal number of MINLP.

Using the optimization problem and general warehouse model defined above, we can state that for each storage area design of each possible solution alternative a MINLP has to be solved. The number of MINLP to be solved can only be estimated, but prevents to find an efficient solution technique. For that reason, we decided for a different approach: For problems without efficient exact solution techniques, heuristics can be used.

Although they are only "second best" approaches [5], the important advantage of heuristics for our problem is the possibility to build a flexible and adapted algorithm.

## 6 The Heuristic Solution Technique

The heuristic solution technique aims to find a solution for the problem defined by (3-1) to (3-5). Each solution alternative corresponds to a warehouse consisting of storage areas. All goods are assigned to one of the storage areas. Each storage area is designed in a way that enables all assigned goods to be stored and all orders to be fulfilled.

The idea behind the solution technique is to start with an empty warehouse and to store all goods of the given assortment using a stepwise approach. The warehouse thus "grows" with each planning step and adapts itself to make the new goods fit in. With each planning step, we consider one article or a group of goods. The result of each planning step is used as the initial solution alternative for the next one. We then apply the second and third step of planning as described in [5]: From the initial solution alternatives, new ones are derived using two mechanisms: On the one hand, a new storage area can be added to the initial solution alternative, which only contains the goods of the current planning step. On the other hand, an already existing storage area can be extended to fit in the goods of the current planning step (see Figure 4). Next, the solution alternatives are evaluated using the general warehouse model. By comparing the costs of the initial and derived solution alternatives, we choose the solution alternative with the lowest increase in costs as the result.


Figure 4: Mechanisms to Derive New Solution Alternatives
The solution technique can be presented as pseudo-code as follows:
Function SolutionTechnique()
\{
warehouse $\mathrm{x}_{\mathrm{init}}=$ new warehouse();
planningsteps $=$ GroupGoodsToPlanningSteps();

```
    foreach (var ps in planningsteps) {
        X Xdd = CreateSolutions_ModeAdd(x (}\mp@subsup{\textrm{x}}{\mathrm{ init,}}{
        XExt = CreateSolutions_ModeExt(xinit, ps);
        foreach (var wh in }\mp@subsup{X}{\mathrm{ Add }}{}\mathrm{ ) { wh.EvaluateCosts(); }
        foreach (var wh in X Xet) { wh.EvaluateCosts(); }
        X Xdd.Sort(\Delta, up);
        X Ext.Sort(\Delta, up);
        if (X
        else { X Xinit = X Xext[0];}
    }
}
```

In the pseudo code, we use a function CreateSolutions_ModeAdd() which creates a set of solution alternatives by adding a new storage area for the goods of the current planning step according to mechanism 1 in Figure 4. The function CreateSolutions_ModeExt() complies with mechanism 2 in Figure 4, creating a set of solution alternatives by assigning current goods to different existing storage areas and by extending these storage areas to make the newly assigned goods fit.

For the evaluation of costs using (4-1) and the general warehouse model, each solution alternative contains a function EvaluateCosts(). Result from EvaluateCosts() is the increase in costs $\Delta$ compared to the initial solution. This increase is used to chose the best solution alternative for the next planning step.

There are several ways to adapt the heuristic solution technique. For example, instead of a single initial solution alternative $\mathrm{X}_{\text {init }}$, a set of solution alternatives $\mathrm{X}_{\text {init }}$ can be transferred from one planning step to the next. This adaption leads to broader set of solution alternatives to be generated in each step, as they are derived from different initial solution alternatives in $\mathrm{X}_{\text {init. }}$. A second possible way to adapt the solution technique is to vary the order of planning steps. Besides a random order of planning steps with or without repetitions, planning steps can be arranged using article properties such as the required number of storage spaces or load unit dimensions.

## 7 Findings From Applying the Solution Technique

As the quality of the solution alternatives provided by heuristic approaches is unknown, we focus on the applicability of the solution technique to industrial practice. To evaluate it therefore means we have to prove it to be applicable and to provide solution alternatives of sufficient quality fast enough.

To prove the applicability, we considered data from a unit load warehouse operated by a logistics service provider. The assortment of about 1,500 goods was classified based on load unit height and mean stock (see Table 1). We considered three types of S/RV with two operational heights and according prices each: forklifts $(5 \mathrm{~m} / 8.5 \mathrm{~m}$, $25,000 € / 30,000 €$ ), reach trucks ( $7 \mathrm{~m} / 13 \mathrm{~m}, 27,500 € / 35,000 €$ ), and high shelf stackers ( $7 \mathrm{~m} / 13 \mathrm{~m}, 60,000 € / 65,000 €$ ). Pallet rack and ground storage can be chosen as storage
equipment (at $35 € / 0 €$ per storage space). The costs taken into account consist of costs for storage equipment, $\mathrm{S} / \mathrm{RV}$, and area (at $200 €$ per $\mathrm{m}^{2}$ ).

Table 1: Classified Assortment from Industrial Practice

| ID | Number of <br> Goods [-] | Load Unit <br> Footprint [mm²] | Load Unit <br> Height [mm] | Max. Stack <br> Height [-] | Mean Stock <br> per Article [-] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Low_LU_ <br> High_Stock | 50 | $1,200 \times 800$ | 540 | 3 | 12 |
| Low_LU_- <br> Low_Stock | 1367 | $1,200 \times 800$ | 540 | 3 | 1 |
| High_LU_ | 19 | $1,200 \times 800$ | 1,800 | 3 | 24 |
| High_Stock <br> High_LU_ <br> Low_Stock | 43 | $1,200 \times 800$ | 1,800 | 3 | 1.5 |

Each class of goods determines one planning step. For each step, the solution technique creates and evaluates between 100 and 1,200 solution alternatives in an elapsed time of three seconds, using a customary computer. In the following, we focus on the assignment of goods and equipment selection as they provide a fundamental idea of the storage area design. However, the created solution alternatives differ in all possible variables from design and strategies, e.g. in the orientation of aisles or in storage space size. Among the 10 best solution alternatives, the running costs per year are nearly identical with less than $1 \%$ difference. Although it is possible to store all goods in the same storage area, these 10 solution alternatives consist of 3 or 4 storage areas. Table 2 shows that only 4 combinations of S/RV type and storage equipment occur.

Table 2: Occurring Storage Area Types

| ID | Storage Equipment | S/RV Type |
| :---: | :---: | :---: |
| SA1 | Pallet Rack | Reach Truck (7 m) |
| SA2 | Pallet Rack | Reach Truck $(13 \mathrm{~m})$ |
| SA3 | Pallet Rack | High Shelf Stacker $(13 \mathrm{~m})$ |
| SA4 | Ground Storage | Reach Truck $(7 \mathrm{~m})$ |

Table 3 shows how often each article class is stored in which storage area type from Table 2. "Low_LU_Low_Stock" in the second column for example is stored twice in SA1, and 8 times in SA2.

Table 3: Assignment of Article classes to Storage Area Types

|  | Low_LU_ <br> High_Stock | Low_LU_-_Stock <br> Low_Stock | High_LU_ <br> High_Stock | High_LU_- <br> Low_Stock |
| :---: | :---: | :---: | :---: | :---: |
| SA1 | 0 | 2 | 0 | 2 |
| SA2 | 3 | 8 | 0 | 3 |
| SA3 | 3 | 0 | 0 | 5 |
| SA4 | 4 | 0 | 10 | 0 |

We can sum up the content of Table 3 with the following findings, additionally considering the assignment of goods to storage areas:

- A maximum of 2 article classes are stored together. Only 2 combinations occur: Low_LU_High_Stock with Low_LU_Low_Stock or High_LU_High_Stock.
- High_LU_High_Stock contains the lowest number of goods and the highest mean stock per article. It is in all cases stored in a ground storage system. This complies with the general statement about the usage of ground storage, e.g. in [1; 9].
- Low_LU_Low_Stock contains the largest number of goods and the least mean stock per article. It is in all cases stored using a pallet rack. This complies with the general statement about the usage of ground storage, e.g. in [1; 9].
- High_LU_Low_Stock is in all cases stored in a separate storage area using pallet racks. Because of the low stock per article, stacking load units in a ground storage is not beneficial.
- Low_LU_High_Stock is stored using pallet racks or ground storage. If stored separately ( 3 of 10 cases), a pallet rack with high shelf stackers is used. If stored with the classes Low_LU_Low_Stock (3/10) or High_LU_High_Stock (4/10), the storage equipment and S/RV are determined by those classes. Thus in some cases, it is beneficial to extend other storage areas with Low_LU_High_Stock instead of storing this class separately.
Due to proprietary reasons, we are not able to provide a comparison of the costs of the real system to the costs of the solution alternatives we created. But, the findings from applying the solution technique show that it is not always clear which storage area design and article assignment results in the lowest costs. Thus, deviating from general statements about the best assignment of goods can be beneficial under certain circumstances. Accordingly, the more important is it for planners to know about solution alternatives and their potential. Based on these results, promising solution alternatives can be identified to focus on during the detailed planning stage.

In addition to applying the solution technique to an example from industrial practice, we invited experienced planners to work on sample planning tasks using the application. In an online survey, we asked the participants afterwards about their assessment of the applicability to gross planning in industrial practice. From a total of 19 participants, we received 12 surveys. The participants affirmed the approach of being fast and providing support during the gross planning phase. In addition, the effort needed to collect and enter data is confirmed to be low.

## 8 Conclusion

The paper at hand addresses the problems of sizing and dimensioning, storage area layout, equipment selection, operation strategy, and assignment of goods to storage areas when planning manually operated storage/retrieval systems. We use running costs for area, construction, storage equipment, storage/retrieval vehicles and employees as a measure of the quality of a solution alternative: The lower the costs generated by a solution alternative, the better it is. A solution alternative corresponds with a warehouse consisting of m storage areas. We give a formulation on the optimization problem of finding the best solution alternative, while the design of each storage area as well as the assignment of goods to storage areas are considered as variables.

To be able to evaluate the solution alternatives, we provide a short description of the general warehouse model we use. Its tasks are to calculate the geometry and cycle time of storage areas for different sets of design variables in the scope. Inserting the warehouse model into the target function of our previously stated optimization problem leads to the conclusion that no efficient solution technique can be found to solve the problem exactly. The main reason for this is that there are variables that do not occur as numeric values in the objective function, but on the contrary define its shape.

Due to this complexity, we present a heuristic approach to solving the optimization problem. The idea behind the approach is to start with an empty warehouse and then add more goods with each planning step. Article assignment and storage area design are considered simultaneously as we generate new solutions using two mechanisms.

We want to present a solution technique to support planners during the rough planning phase. During this planning step, it is important for planners to get a quick overview of possible solution alternatives. By implementing our solution technique and applying it to an example from industrial practice, we show that our heuristic approach is fast and provides solution alternatives with different storage area designs, and article assignments. Planners can use these results to decide which solution alternatives to focus on.

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