

# INTEGRATED VERSUS SEQUENTIAL SCHEDULING AND ASSIGNMENT AT A UNIT LOAD CROSS-DOCK

Arpan Rijal

Rotterdam School of Management, Erasmus University, Rotterdam,  
The Netherlands

Marco Bijvank

Haskayne School of Business, University of Calgary, Alberta, Canada

René de Koster

Rotterdam School of Management, Erasmus University, Rotterdam,  
The Netherlands

## Abstract

Within a cross-dock, the assignment of trucks to dock-doors and the scheduling of trucks to be processed are two major operational decisions. Conventionally, assignment and scheduling decisions are made sequentially. However, solving the two problems sequentially can lead to sub-optimal solutions because the objectives of the two problems are in conflict with each other. To gain further insights, we create an integrated model which is capable of simultaneously scheduling and assigning trucks at cross-docks. We contrast the integrated model with a sequential model which first schedules trucks for processing and then assigns them to dock-doors. Experiments demonstrate that the integrated model can produce superior solutions, despite that it is computationally more expensive.

## 1. Introduction

Cross-docks are facilities that allow the consolidation of shipments from multiple origins to a single destination with minimal storage of shipments within the cross-dock facility [6], [7]. Incoming shipments and their departure are synchronized such that the shipments remain within the cross-dock usually for less than a day and often for less than an hour [6], [7].

The operations at a cross-dock facility require numerous decisions that can be divided into strategic, tactical, and operational levels [7], [16], [22]. Decisions about the shape of the cross-dock, the number of dock-doors, and the means of moving shipments within the cross-dock belong to the strategic level [16]. Service modes of the dock-doors are decided at the tactical level. A dock-door can function as dedicated inbound, dedicated outbound or flexible (i.e., mixed mode) dock-door. Unlike inbound or outbound dock-doors, mixed mode dock-doors are capable of processing both inbound and outbound trucks at different points in time [16]. Bodnar et al. [4] show that a cross-dock with flexible dock-doors can achieve the same operational performance compared to a cross-dock with only dedicated inbound and outbound dock-doors, but with fewer dock-doors. Other tactical level decisions within a cross-dock

concern the use of a temporary storage area and the determination of the amount of personnel and equipment that need to be available at the cross-dock.

At the operational level, the assignment of trucks to dock-doors and the scheduling of trucks to be processed at the dock-doors are the two major decisions. In the assignment problem, trucks are assigned to specific dock-doors with the objective to minimize the internal travel distances within the facility [3], [10], [14], [19], [20]. This is an important objective because cross-docks can be large with significant distances between dock-doors. The reduction of travel distances has a positive impact on productivity by reducing labor and equipment usage, and it increases customer service by reducing the time required to complete the transshipment processes [3]. The assignment of trucks to dock-doors can also be done at a tactical level. In such a case, all trucks originating from or destined to certain locations are assigned to the same dock-door throughout the tactical planning period. In this paper, we only refer to assignment decisions at the operational level.

Cross-dock scheduling involves the sequencing of trucks for processing when the number of trucks is larger than the number of dock-doors. In contrast to the assignment problem, internal travel distances within the cross-dock are not considered in the scheduling problem. Schedules can be created with the objective to minimize delayed shipments [6], lost shipments [17], makespan [2], [8], [21], [25], temporary storage [12], temporary storage and tardiness of outbound trucks [4], or to maximize throughput [18].

The scheduling of inbound and outbound trucks without considering the assignment of trucks to dock-doors would suffice for small cross-dock facilities. This is because the travel times between dock-doors are small. Consequently, the sum of the travel distances from solutions to the assignment problem might not differ much compared to the total travel distance when trucks are assigned to dock-doors on a first-come-first-assigned basis.

However, in larger cross-dock facilities with a high truck-to-dock-door ratio, it could be beneficial to solve the scheduling and assignment problem of trucks in an integrated manner. By doing so, inbound and outbound truck pairs can be docked closer to each other which leads to lower internal travel distances without compromising scheduling objectives.

Scheduling and assignment problems have mostly been tackled sequentially within the cross-docking context because of the inherent complexity in each of the two problems. However, significant gains may be realized by integrating both problems. We develop two models to evaluate the differences between solving the integrated scheduling and assignment problem versus the sequential approach. The first model considers the scheduling and assignment problem in an integrated manner. While, the second model first schedules inbound and outbound trucks and then assigns them to dock-doors. Both models aim to minimize internal travel distances within the cross-dock, delays of outbound trucks, and usage of temporary storage. In addition, we compare both models extensively on randomly generated instances to gain insights into the trade-offs between the integrated and sequential model.

## **2. Literature Review**

Cross-docking has received significant research attention with numerous papers on the topic in recent years. In this review, we only refer to works in the cross-docking literature where both the scheduling and assignment of trucks are studied. For other aspects of cross-docking and related decision problems, we refer to overviews by Agustina et al. [1], Boysen & Flidner [6], Van Belle et al. [22], Buijs et al. [7], and Ladier & Alpan [16].

Chmielewski et al. [9] consider a less-than-load (LTL) truck terminal where inbound shipments are consolidated. Inbound trucks unload items, which are loaded onto outbound trucks. The objective of the problem is to schedule and assign the inbound and outbound trucks such that the travel distance within the truck terminal is minimized as well as the loading and unloading time. Each outbound truck (or destination) is assigned to a unique dock-door on a tactical basis for a period of six months or more. The problem is formulated as a multi-commodity flow problem with side constraints. Solution techniques based on column generation and evolutionary algorithms are also provided.

In a large cross-dock where the number of trucks is larger than the number of available dock-doors, the model presented by Chmielewski et al. [9] is not applicable. Each outbound truck receives a unique dock-door in their model, and the assignment of the outbound truck to the dock-door persists over the entire tactical planning horizon. When there are more trucks than dock-doors there will not be enough dock-doors to assign each truck. Furthermore, the use of a temporary storage area within the cross-dock is also not considered.

Boysen [5] presents a cross-dock scheduling model for a cross-dock with fixed mode dock-doors without temporary storage. The cross-dock of concern is used for the transportation of refrigerated food products. The entire planning period is broken into discrete time periods. Each inbound truck is processed within exactly one time period. Since every inbound truck arrives fully loaded and carries identically sized items, the processing times for all inbound trucks are considered to be deterministic and equal.

The model can be adapted for three objectives: the minimization of flow times for outbound trucks, the minimization of processing times for outbound trucks and the minimization of the tardiness for outbound trucks. Dynamic programming and simulated annealing based algorithms are presented as solution techniques to the problem. The solution techniques are tested on instances with two to five inbound dock-doors and four to eight time periods.

The model presented by Boysen [5] is most suitable for small fresh food cross-docks where the same temperature has to be maintained throughout the supply chain. In contrast, larger cross-docks handle a vast variety of products with different temperature requirements. Using a temporary storage might not be completely prohibited although it is unfavourable because of the prolonged temperature shock. This is especially the case for fresh and ambient food products. Furthermore, the model only considers fixed mode dock-doors. More importantly, internal travel distances are not part of any objectives.

Van Belle et al. [23] present a model that simultaneously assigns and schedules inbound and outbound trucks. The dock-doors operate in fixed modes only. The processing time of a truck is proportional to the number of shipments in the truck. The authors consider a temporary storage area with infinite capacity. Personnel and equipment are sufficiently available for any necessary transfers of items between dock-doors within the cross-dock. The objective is to minimize the sum of internal travel distances and delays in truck departures of both inbound and outbound trucks. The problem is formulated as a mixed integer linear program (MILP) and a tabu search based solution approach is provided. Solutions obtained from the tabu search procedure are then fed into a CPLEX solver to make further improvements.

In the model of Van Belle et al. [23], shipments can be transferred through a temporary storage area without being penalized in the objective function. In reality, transfers through a

storage area incur double handling of shipments and should be included to the objective function [4]. Additionally, flexible dock-doors are not considered by the authors. In all of the numerical tests, the number of inbound and outbound dock-doors is limited to three. These instances are too small to guarantee that the solution approach is applicable to real-sized instances.

Hermel et al. [14] consider a truck terminal with an equal number of trucks and dock-doors. The scheduling and assignment of trucks is done sequentially. First, the inbound and outbound trucks are clustered into disjoint sets according to the flows of materials between them. The trucks are then assigned to dock-doors with the intention to minimize the total travel distance. Clustering inbound and outbound trucks into disjoint sets reduces the complexity to solve the assignment problem of trucks to dock-doors, because the assignment problem can be solved for each set separately. This reduces the problem size and the solution space. Next, the workforce is scheduled as a multi-mode resource constrained project scheduling problem. Finally, the trucks are scheduled according to the workforce plans. The overall objective of the problem is to minimize the sum of travel distances and the makespan.

The solution framework presented by Hermel et al. [14] is only applicable to a cross-dock with an equal or fewer trucks and dock-doors. Furthermore, in the case when all trucks are connected to each other through material flows, the clustering of trucks into small disjoint sets is not possible. If this is the case, the assignment problem has to be solved for the entire problem instance at the same time. Another limitation of this model is that it only considers fixed mode dock-doors.

To the best of our knowledge, no work exists that simultaneously schedules and assigns trucks in cross-docks with flexible dock-doors and temporary storage with the objective to minimize internal travel distances, usage of temporary storage, and delays of outbound trucks. Furthermore, no existing work has investigated the benefit of integrating the scheduling and assignment decisions versus solving both problems sequentially. *Table 1* presents an overview of the most relevant work in the existing literature and our contribution.

Table 1: Comparison of relevant work on truck scheduling and dock-door assignment.

Publications	Cross-dock Features		Objectives		
	Flexible Dock-doors	Temporary Storage	Travel Distance	Outbound Delays	Temporary Storage
Chmielewski et al. (2009)		□	□		
Boysen (2010)				□	
Van Belle et al. (2012)		□	□	□	
Hermel et al. (2016)	□		□		
Our paper	□	□	□	□	□

### 3. Problem Description

We consider a cross-dock that serves a set of inbound and outbound trucks represented by  $I$  and  $O$ , respectively. Each inbound truck contains shipments that are designated to a specific set of outbound trucks. For each unit load in an inbound truck, information about the designated

outbound truck is known beforehand. Interchanging unit loads between outbound trucks is not permitted. The amount of loads transferred between inbound and outbound trucks are presented in a  $|I| \times |O|$  transshipment matrix where entry  $f_{ij}$  indicates the number of unit loads in inbound truck  $i \in I$  destined for outbound truck  $j \in O$ .

The cross-dock has a given number of dock-doors, divided in inbound, outbound, and flexible dock-doors, indicated by the sets  $D_I$ ,  $D_O$ , and  $D_F$  respectively. Only inbound trucks can be unloaded at inbound dock-doors, and only outbound trucks can be loaded at outbound dock-doors. However, flexible dock-doors can be used for both unloading of inbound trucks and loading of outbound trucks. Once an inbound truck  $i \in I$  is assigned to a flexible dock-door, the dock-door serves as an inbound dock-door until the truck leaves, i.e., all inbound unit loads from the truck are unloaded, after which the dock-door can be used again as an inbound or outbound dock-door. Likewise, a flexible dock-door assigned to an outbound truck  $j \in O$  is designated as outbound dock-door between the docking and departure of the fully loaded outbound truck.

Only when both inbound and outbound trucks are docked at the same time, direct dock-to-dock transfers are possible between them. Otherwise, if an inbound truck  $i \in I$  is unloaded and departs before outbound truck  $j \in O$  is docked where  $f_{ij} > 0$ , the unit loads  $f_{ij}$  have to be routed via the temporary storage area.

The usage of the temporary storage area incurs cost  $\eta$  per unit load which includes the cost of internal transportation and additional handling. Numerous storage policies can be employed to store unit loads temporarily at the temporary storage areas. Provided that the assignment of inbound and outbound trucks to dock-doors is given, Vis & Roodbergen [24] provide a methodology to choose the optimal location to temporarily store shipments. However, the method becomes inapplicable when the assignment of inbound and outbound trucks to dock-doors is part of the optimization problem. Furthermore, we observe that shipments are stored using the closest open location storage in unit-load cross-docking, i.e., the unit loads are stored in the nearest empty location. Internal travel distances to and from the temporary storage area and the personnel and equipment cost of double handling of unit loads using such temporary storage policy can be approximated using the random storage policy, where the probability of a shipment being stored temporarily in any of the locations in the temporary storage area is equally likely [11].

The distance between two dock-doors,  $k$  and  $p$  is indicated by  $m_{kp}$ . In case of a direct dock-to-dock transfer, each unit load accrues cost  $\beta$  which includes the cost of handling and transporting unit loads over a unit distance.  $\beta$  can be approximated by considering the personnel and equipment time taken to handle and transport each unit load over a unit distance.

The planning horizon is divided into a set of  $T$  time periods of equal length. The length of each time period is determined by the expected time required to process and unload inbound trucks [5]. In unit load cross-docking with evenly filled inbound trucks, the amount of time required to process and unload inbound trucks is approximately the same [5]. Furthermore, we assume that the cross-dock facility has enough personnel and equipment to perform all transshipments within one time period.

The arrival and departure times of a truck  $i \in I \cup O$  are given and denoted in terms of time windows by  $r_i$  and  $d_i$ , respectively.  $r_i$  and  $d_i$  coincide with the beginning or the end of the discrete time periods. The cross-dock operator has information on the arrival and departure period of each truck. Since we consider a static problem, any deviations in arrival times can have an impact on the feasibility of our solutions in practice. Historical arrival data can be used to estimate the deviation in arrival times in order to create robust time windows [4].

Additionally, some slack can be built into the time windows to ensure that the trucks arrive at the beginning of the arrival time period.

Inbound and outbound trucks cannot be processed before their arrival period. Inbound trucks have to be unloaded completely within the given time window. However, the loading of an outbound truck can be delayed beyond its due time while accruing tardiness cost, denoted by  $\gamma$ , for each time period the outbound truck is delayed. Particularly in retail distribution, outbound delays can be expensive because of tight time window restrictions for the delivery at retail stores. Outbound delay cost  $\gamma$  can be estimated by considering the additional cost of the outbound truck and truck driver, and the cost at the retail stores for not respecting the time window. The latter can be represented by the cost of additional staff to unload the unit loads or penalties payable to city authorities for the breach of time windows.

Each inbound and outbound truck has to remain docked for a minimum of  $g$  and  $h$  time intervals, respectively. These minimum docking times represent the amount of time that is required to dock a truck, perform load transfers and administrative processing activities at the dock-door.

We assume that internal resources within the cross-dock are infinite. We also assume that enough space is available in the cross-dock to avoid congestion. Preemption of trucks is not allowed, i.e., once trucks are docked they must be fully loaded or unloaded at the same dock-door before departing.

The problem description outlined here is largely inspired by the cross-dock operations of a Dutch retailer. However, it can easily be extended to any large cross-dock with a high truck-to-dock-door ratio.

#### 4. Integrated Model

We create an integrated integer programming model which jointly assigns and schedules both inbound and outbound trucks to dock-doors. The objective of the model is to minimize the sum of the weighted delays of outbound trucks, direct travel distances between dock-doors, and the usage of temporary storage indicated by the travel distance between the dock-doors and the temporary storage area. The weights in the model are  $\beta$ ,  $\eta$  and  $\gamma$ , which indicate the cost of per unit travel distance per unit load, cost of usage of temporary storage per unit load and the cost of the delay per outbound truck per time period, respectively.

In this time discrete model, the main decision variables are binary and indicate at what time period and at which dock-door a particular truck is docked. Other auxiliary variables indicate the completion time of the (un)loading of each truck and the number of (un)loaded unit loads for each truck at each time period. Additionally, the model also includes variables that indicate for each time period the number of units at the temporary storage area that are unloaded from a particular inbound truck and is destined to be loaded to a particular outbound truck.

The constraints in the model are the same as outlined in the problem description. In brief, the constraints ensure that (i) each truck is assigned to only one dock-door; (ii) a dock-door at any particular time period is only assigned to one truck; (iii) inbound and outbound trucks are only assigned to appropriate dock-doors; (iv) inbound and outbound trucks can only be processed after their arrival; (v) within the planning horizon each inbound and outbound truck has to be completely unloaded or loaded, respectively; (vi) direct dock-to-dock transfers are only possible if both inbound and outbound trucks with unit loads flow between them (i.e.,  $f_{ij} > 0$ ) are docked at the same time period; (vii) unit loads have to be sent to the temporary

storage area if the outbound truck is not docked while the inbound truck is scheduled to depart in the next time period; (viii) all units in the storage area need to be loaded before the end of the planning horizon. It is possible that some unit loads from an inbound truck can be sent directly to outbound trucks that are docked at the same time period as the inbound truck and the remaining unit loads that are designated to be loaded onto outbound trucks that are not docked while the inbound truck is docked have to go to the temporary storage area; (ix) each truck has to be processed for a minimum number of time periods to account for administrative and transportation activities; and (x) no preemption of trucks is allowed.

Some of the decision variables in the integrated model interact with each other non-linearly. Therefore, in addition to the original integrated model, we need to create a linear version by introducing additional variables which capture the same interaction as the decision variables in the non-linear formulation.

## **5. Sequential Scheduling and Assignment**

The sequential approach decomposes the integrated problem into distinct scheduling and assignment problems. Firstly, we create an integer programming model that creates schedules for inbound and outbound trucks. Secondly, an assignment model assigns trucks to dock-doors while the schedule when the trucks need to be docked is given.

### **5.1 Scheduling Problem**

The first model schedules when all inbound and outbound trucks need to be docked. The objective function in this stage only captures the scheduling objectives of the integrated model, which is to minimize the delay of outbound trucks and the usage of the temporary storage area.

The main variables in the model represent the times when each inbound (outbound) truck starts and ends unloading (loading). Compared to the decision variables in the integrated model, these variables do not have an additional index that specifies the docking location of the truck. This drastically reduces the number of variables in the model. Other auxiliary variables track the number of loaded and unloaded unit loads for each inbound and outbound truck at each time period and the number of unit loads between trucks at the temporary storage area. Similar to the integrated model, direct shipments are only possible when both the inbound truck and the outbound truck with unit load flows between them are docked at the same period. If an inbound truck departs before the outbound truck arrives that should receive unit loads from this inbound truck, then the designated unit loads need to be temporarily stored until the outbound truck is docked.

The assumptions and constraints are equivalent to those in the integrated model. However, their formulations are less involved. Namely, the number of constraints and variables is reduced as we do not track the exact docking position of each truck. The original formulation of the scheduling model suffers from the same non-linear formulations as Section 4, which we address by introducing additional linearizing variables.

### **5.2. Assignment Problem**

The schedule that is generated by the scheduling model is used as input for the assignment stage. The assignment model allocates trucks to dock-doors with the objective to minimize the weighted travel distances of direct dock-to-dock transfers between inbound and outbound trucks.

The decision variables are binary and indicate whether a certain inbound or outbound truck is docked at each particular dock-door. The constraints in the model ensure that each truck is assigned to an appropriate dock-door, i.e., inbound, outbound or flexible dock-door, when the truck needs to be docked according to the schedule. Furthermore, the constraints in the model also ensure that once a truck is assigned to a dock-door it has to remain there for the time periods as indicated by the schedule.

## 6. Computational experiments

The integrated model and the sequential model are implemented in *Gurobi 6.0.3* through *Python* interface on an i7-3520M CPU 2.9 GHz machine with 8 GB RAM. The integrated model as well as each of the sequential models are run for a maximum of 1,800 seconds. For consistency, all models are run with one CPU thread. The creation of the test instances is explained in Section 6.1 and the actual numerical results in Section 6.2.

### 6.1. Instances

The instances for the computational experiments are created according to the approach taken by Konur & Golias [15] and Gue [13]. Bodnar et al. [4] do the same. The planning horizon has a length of 12 hours and is divided in 16 intervals of 45 minutes.

#### 6.1.1. Inbound and outbound trucks

The number of inbound and outbound trucks in each instance is the same, i.e.,  $|I| = |O|$ . We limit the number of inbound and outbound trucks in the instances to only 10 trucks to increase the chances that both models find an optimal solution such that we can compare the outcome of the two models.

We set the difference between the arrival and departure time of each truck, i.e. the time window, to either 2 or 3 periods to test the impact of the length of these time windows. The actual start and end time of each window is randomly generated as suggested by Konur and Golias [15]. A midpoint for the time window of each truck is randomly chosen from the discrete uniform  $U[1,16]$  distribution. Next,  $r_i$  and  $d_i$  are set such that the average of the two values equals the midpoint and the difference equals the given length of the time window. If  $r_i < 1$  or  $d_i > 16$ , the arrival and departure windows are adjusted accordingly to fall within the planning horizon.

The transshipment matrix between inbound and outbound trucks is generated according to the procedure proposed by Gue [13]. The number of outbound destinations for each inbound truck is chosen randomly from a discrete uniform distributions  $U[3,5]$  or  $U[5,7]$ , i.e., the inbound trucks supply unit loads to 4 or 6 outbound trucks on average. The outbound trucks that receive the unit loads from the inbound truck are chosen randomly. The number of unit loads in each truck is set to 24, which is the number of standard Western European pallets in a trailer load [4]. To ensure that each inbound and outbound truck has 24 pallets each, we solve a MILP problem for each  $|I| = |O|$  transshipment matrix. The problem allocates values to the cells in the transshipment matrix such that the sum of each row and column equals 24 and the cells that should have non-zero entries are the same as determined randomly for each inbound truck using the given probability distributions.

As by Bodnar et al. [4], we generate the transshipment matrices and truck schedules independently. Furthermore, we consider that the minimum processing time for all inbound and outbound trucks equals one time period.



### 6.1.2. Dock-door layout

We consider a ‘U’-shaped cross-dock with either 5 or 15 dock-doors on the same side of the facility. Smaller cross-docks with fewer dock-doors exist in practice. However, the impact of internal travel distances can be more profound in bigger cross-docks. Gue [13] reports cross-docks with 300 dock-doors. In such large cross-docks, the impact of travel distances can be enormous and the differences between the solutions of the integrated and sequential model can be significant. However, we can only expect that both models converge to optimal solutions for small cross-docks. Next, the mode of each dock-door needs to be selected. Each door can be dedicated to either inbound or outbound trucks or it can be a flexible dock-door. A large number of layout combinations are possible even with only five or fifteen dock-doors. To gain some insights, we consider a limited number of configurations for the experiments.

In the instances with 15 dock-doors, the first layout, *L1*, is based on a cross-dock facility of a Dutch retailer. The three dock-doors in the middle (i.e., dock-doors 7 until 9) operate in a dedicated inbound mode and the remaining dock-doors on either side operate as outbound dock-doors. In the second configuration, *L2*, the third outbound dock-door is changed to a flexible dock-door. In the third layout, *L3*, one additional flexible dock-door is introduced compared to *L2*.

*L4*, *L5*, and *L6* represent the dock-door layout in the cases with 5 dock-doors. In *L4*, the center dock-door is an inbound dock-door and the other two dock-doors on either side are outbound dock-doors. *L5* replaces one of the outbound dock-doors next to the inbound door of *L4* to a flexible dock-door. Finally, in *L6*, both outbound dock-doors on either side of the inbound dock-door of *L4* are replaced by flexible dock-doors. Table 2 gives an overview of the six different layout configurations.

The distance between the adjacent dock-doors in each layout is 3.6 meters, which is the length of 3 pallets. We disregard the distance from the truck to the staging lane, since this part of the internal transportation is constant for all movements of unit loads regardless of any solution.

Table 2: Dock-door layouts.

Configuration	Layout*
<b>L1</b>	o o o o o o i i i o o o o o
<b>L2</b>	o o f o o o i i i o o o o o
<b>L3</b>	o o f o o o i i i o o f o o
<b>L4</b>	o o i o o
<b>L5</b>	o o i f o
<b>L6</b>	o f i f o

\* i - inbound dock-door, o - outbound dock-door, f - flexible dock-door

### 6.1.3. Weighted Costs

The weights for the direct dock-to-dock travel, usage of temporary storage and delays of outbound trucks can be estimated as described in Section 3. However, the trade-offs between the sequential model and the integrated model in terms of computation time and solution quality can be different for different weights. To analyze these trade-offs, we consider different combinations of weights for each of the instances we solve:

- $\beta \in \{1\}$
- $\eta \in \{10, 50\}$
- $\gamma \in \{100, 500\}$

## 6.2. Numerical results

The performance of the integrated and sequential model is shown in Table 3 and 4 for the instances with 5 and 15 dock-doors, respectively. The results show that the integrated model can produce better solutions compared to the sequential model. Among the 96 instances we test, the solutions generated by the integrated model outperform the sequential model by as much as 6.5 percent. On average, the integrated solution is 0.52 percent better than the sequential solution. However, the integrated model was not able to converge within 1,800 seconds for 44 instances. In 7 instances, the sequential model outperformed the integrated model within the runtime limit. In the worst case, the integrated solution was 11.6 percent worse than the sequential solution found within the time limit.

For the 48 instances we tested with 5 dock-doors, shown in Table 3, the integrated solution outperforms the sequential solution by at most 6.5 percent. In 33 of the 48 instances some savings are realized. However, the majority of integrated solutions, 24 out of 48, only differ around 1 percent with the sequential solutions. In 12 instances, the integrated model is not able to converge to optimality within 1,800 seconds.

When the number of dock-doors is increased to 15, the largest savings is 4.3 percent, as indicated in Table 4. In these 48 instances, the integrated model produces optimal solution within the time limit on 16 occasions only. As the number of dock-doors increases, the problem size increases and the model requires more time to converge to an optimal solution.

When the ratio of the direct travel cost between the dock-doors is higher,  $(\beta/\eta)/(\gamma) = (1/10)/100$ , the integrated model produces much better solutions compared to the sequential model. With these ratios, direct dock-to-dock transfers become more relevant and it could be better to postpone the processing of outbound trucks, possibly even with delays, such that inbound and outbound trucks can be assigned closer together. The highest savings generated by the integrated model for this ratio is 4.3 percent and the average savings is 1.99 percent.

In contrast, with  $(\beta/\eta)/(\gamma) = (1/50)/500$ , indicating that the direct dock-to-dock transfer is significantly cheaper compared to the usage of the temporary storage area or delays of outbound trucks, the integrated solutions do not differ significantly from sequential solutions. The highest savings in this case is 1 percent and the average savings is 0.3 percent. When the direct dock-to-dock travel is significantly cheaper compared to the cost of temporary storage and delays of outbound trucks, scheduling objectives are more important than the assignment objectives. Therefore, the additional gains that can be made by using the integrated model can be insignificant.

In all instances, the integrated model requires more time to find the optimal solution compared to the sequential model. The integrated model has significantly more variables and constraints. As a result, the solver requires significantly more time to generate good solutions.

The sequential model converges in 91 of the 96 instances. In contrast, the integrated model produces optimal solutions for only 52 instances.

The impact of the length of time windows and the average number of outbound trucks per inbound truck is discernible. The integrated and sequential models take longer to converge when the length of the time window increases or when the average number of outbound trucks per inbound truck increases. In the former case, the solver requires more time to parse through more feasible solutions. In the latter case, the increase in run times can be explained by the additional flows that the model has to handle. These results are consistent with previous observations in the literature [4].

As the number of flexible dock-doors increases, the sequential model and the integrated model produce superior results. However, they require more time to converge. Similarly, the quality of the solutions for both models also increases with additional flexible dock-doors. These dock-doors provide additional flexibility and help remedy bottle necks during periods of peak demand for (un)loading.

## 7. Conclusion

In this paper, we present two models to schedule and assign inbound and outbound trucks in a cross-dock. The first model solves both problems in an integrated manner while the second sequential model decomposes the problem first into a scheduling problem and then into an assignment problem.

Additionally, we test both models for a number of random test instances. The results indicate that the integrated model can provide superior solutions compared to the sequential model, at the expense of larger computational requirements. Depending on the context, integrated models might not offer significant gains compared to the sequential model. If the cost of a direct dock-to-dock transfer of unit loads is significantly cheaper than the usage of a temporary storage area and the delay cost of outbound trucks, then most of the realizable savings can be achieved by solving the scheduling problem of the sequential model. The additional savings that can be made by solving the integrated model can be very little and unjustified in the face of large computation times. However, with an increase in the direct dock-to-dock transfer costs, integrated solutions consistently outperform sequential solutions. Delaying the processing of outbound trucks can facilitate better assignments of inbound and outbound trucks which is not possible with the sequential model. As a result, larger savings can be realized with the integrated model.

The savings from the integrated model could be significant for heavily utilized, large cross-docks with numerous flexible dock-doors. However, each of these features increase the complexity of the integrated model significantly. For real sized problems and operational applications, commercial solvers cannot be expected to produce good solutions within a reasonable computation time. Therefore, it is worthwhile to adopt efficient heuristics to the integrated model.

The next phase of this research will adopt computationally efficient metaheuristics for the integrated model to tackle real sized instances. Additionally, as a case study, the solution methodology will be applied to the cross-dock operations of a retailer. Before we can make methodological recommendations to cross-dock operators, this case study will test the validity and applicability of the model and the solution quality of the heuristics.

Table 3 Experiment results with 5 dock-doors.

t.w.	$\beta = 1, \eta = 10, \gamma = 100$				$\beta = 1, \eta = 50, \gamma = 100$				$\beta = 1, \eta = 10, \gamma = 500$				$\beta = 1, \eta = 50, \gamma = 500$			
	$d_i - r_i = 2$				$d_i - r_i = 2$				$d_i - r_i = 2$				$d_i - r_i = 2$			
layout	Integrated		Sequential		Integrated		Sequential		Integrated		Sequential		Integrated		Sequential	
	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time
(a) no. of trucks = 20, average number of destinations per inbound truck = 4																
L4	3857	61.38	2.1	1.94	5897	177	0.0	3.75	14257	10	0.6	0.46	16297	75.78	0.0	0.96
L5	3612	142.83	5.0	13.47	5493	1419	2.7	38.31	13216	19.71	0.9	14.45	15493	153.42	0.1	13.57
L6	3592	153.45	6.5	19.77	5439	717	1.6	54	13196	56.16	0.5	18.23	15439	171.29	0.2	52.76
(b) no. of trucks = 20, average number of destinations per inbound truck = 6																
L4	3758	51.53	0.4	2.48	6718	117	0.2	8.62	14158	13.97	0.0	1.39	17118	43.78	0.0	2.4
L5	3625	184.84	0.0	29.18	6465	1045	0.1	70.19	13625	137.72	0.1	28.7	16465	114.74	0.0	33.01
L6	3625	358.68	0.0	60.93	6465	1428	0.0	61.19	13625	379	0.0	16.56	16465	133.98	0.1	33.82

t.w.	$d_i - r_i = 3$				$d_i - r_i = 3$				$d_i - r_i = 3$				$d_i - r_i = 3$			
	Integrated		Sequential		Integrated		Sequential		Integrated		Sequential		Integrated		Sequential	
layout	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time
	(a) no. of trucks = 20, average number of destinations per inbound truck = 4															
L4	4332	308.86	1.8	7.78	6732	288	1.1	2.91	16332	87.28	0.5	2.99	18732	169.68	0.4	2.4
L5	4045	>1800	0.6	982.18	6824	>1800	-8.2	>1800	14439	459.02	0.2	219.7	17837	>1800	0.3	594.16
L6	3905	>1800	2.0	1071.56	7067	>1800	-10.6	1287	14305	792.34	0.3	135.62	17346	1660.26	0.7	543.97
(b) no. of trucks = 20, average number of destinations per inbound truck = 6																
L4	3600	416.08	0.0	9.99	6000	835.8	0.0	9.23	13600	4.02	0.0	4.02	16000	1173.19	0.0	9.38
L5	3088	>1800	1.0	1541.19	6721	>1800	-11.6	>1800	11088	171.99	0.1	171.99	13728	>1800	0.29	602.04
L6	3055	>1800	0.2	1800.49	5932	>1800	-6.0	>1800	11046	451.37	0.3	451.37	13486	>1800	-0.2	>1800

t.w. indicates the length of the time windows

obj. indicates the value of the objective

>1800 indicates that the solver has terminated because of the time limit

$\Delta(\%)$  indicates the relative difference between the integrated and sequential solutions

Table 4 Experiment results with 15 dock-doors.

t.w.	$\beta = 1, \eta = 10, \gamma = 100$				$\beta = 1, \eta = 50, \gamma = 100$				$\beta = 1, \eta = 10, \gamma = 500$				$\beta = 1, \eta = 50, \gamma = 500$			
	$d_i - r_i = 2$				$d_i - r_i = 2$				$d_i - r_i = 2$				$d_i - r_i = 2$			
layout	Integrated		Sequential		Integrated		Sequential		Integrated		Sequential		Integrated		Sequential	
	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time
(a) no. of trucks = 20, average number of destinations per inbound truck = 4																
L1	3383	>1800	4.3	4.34	4877	246	2.7	2.80	12583	675.8	1.2	2.69	14077	133	1.0	2.70
L2	3333	>1800	2.4	17.89	4820	302	1.5	16.90	12533	375.94	0.6	16.79	14020	611	0.5	21.74
L3	3320	>1800	1.1	19.03	4807	376	0.6	17.97	12520	551.23	0.3	17.90	14007	719	0.2	39.74
(b) no. of trucks = 20, average number of destinations per inbound truck = 6																
L1	3296	>1800	2.5	8.02	5229	424	1.4	424.04	12484	>1800	0.8	3.13	14429	427	0.5	5.05
L2	3273	>1800	2.4	11.51	5193	1211	1.5	1210.61	12466	>1800	0.7	9.07	14393	1105	0.5	11.73
L3	3282	>1800	1.8	255.96	5192	1800	1.3	>1800	12473	>1800	0.5	189.41	14392	1513	0.4	184.98

t.w.	$d_i - r_i = 3$				$d_i - r_i = 3$				$d_i - r_i = 3$				$d_i - r_i = 3$			
	Integrated		Sequential		Integrated		Sequential		Integrated		Sequential		Integrated		Sequential	
layout	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time	obj.	time	$\Delta(\%)$	time
(a) no. of trucks = 20, average number of destinations per inbound truck = 4																
L1	3649	>1800	2.2	8.02	3863	1479	2.8	5.63	13721	>1800	0.7	4.28	14263	705	0.6	2.91
L2	3567	>1800	2.8	8.81	3860	>1800	0.4	17.99	13189	>1800	0.7	8.81	14267	>1800	0.1	9.45
L3	3435	>1800	3.3	32.26	3678	>1800	1.8	62.26	13081	>1800	0.3	19.30	13660	>1800	0.8	15.90
(b) no. of trucks = 20, average number of destinations per inbound truck = 6																
L1	2674	>1800	2.2	9.52	3123	>1800	-0.2	5.68	9895	>1800	0.4	7.31	10667	>1800	0.5	5.86
L2	2658	>1800	0.9	72.04	3080	>1800	0.6	35.17	9847	>1800	0.3	110.87	10362	>1800	-0.01	111.67
L3	2633	>1800	2.1	71.56	2917	>1800	0.4	33.90	9842	>1800	0.5	49.18	10119	>1800	0.5	37.95

t.w. indicates the length of the time windows

obj. indicates the objective

>1800 indicates that the solver has terminated because of the time limit

$\Delta(\%)$  indicates the relative difference between the integrated and sequential solutions

## References

- [1] Agustina, D., Lee, C. and Piplani, R., “A review: Mathematical models for cross docking planning,” *International Journal of Engineering Business Management*, 2, 2, 47–54 (2010).
- [2] Arabani, A. B., Ghomi, S. F. and Zandieh, M., “Meta-heuristics implementation for scheduling of trucks in a cross-docking system with temporary storage,” *Expert Systems with Applications*, 38, 3, 1964–1979 (2011).
- [3] Bermudez, R. and Cole, M. H., *A genetic algorithm approach to door assignments in breakbulk terminals*, Technical report, University of Arkansas, Mack-Blackwell National Rural Transportation Study Center (2001).
- [4] Bodnar, P., Azadeh, K. and De Koster R., “Scheduling trucks in a cross-dock with mixed service mode dock doors,” Forthcoming in *Transportation Science*.
- [5] Boysen, N., “Truck scheduling at zero-inventory cross docking terminals,” *Computers & Operations Research*, 37, 1, 32–41 (2010).
- [6] Boysen, N. and Fliedner, M., “Cross dock scheduling: Classification, literature review and research agenda,” *Omega*, 38, 6, 413–422 (2010).
- [7] Buijs, P., Vis, I. F. and Carlo, H. J., “Synchronization in cross-docking networks: A research classification and framework,” *European Journal of Operational Research*, 239, 3, 593–608 (2014).
- [8] Chen, F. and Song, K., “Minimizing makespan in two-stage hybrid cross docking scheduling problem,” *Computers & Operations Research*, 36, 6, 2066–2073 (2009).
- [9] Chmielewski, A., Naujoks, B., Janas, M. and Clausen, U., “Optimizing the door assignment in ltl-terminals,” *Transportation Science*, 43, 2, 198–210 (2009).
- [1] Cohen, Y. and Keren, B., “Trailer to door assignment in a synchronous cross-dock operation,” *International Journal of Logistics Systems and Management*, 5, 5, 574–590 (2009).
- [2] De Koster, R., Le-Duc, T., and Roodbergen, K. J., “Design and control of warehouse order picking: A literature review,” *European Journal of Operational Research*, 182, 2, 481-501 (2007).
- [3] Forouharfard, S. and Zandieh, M., “An imperialist competitive algorithm to schedule of receiving and shipping trucks in cross-docking systems,” *The International Journal of Advanced Manufacturing Technology*, 51, 9-12, 1179–1193 (2010).
- [4] Gue, K. R., “The effects of trailer scheduling on the layout of freight terminals,” *Transportation Science*, 33, 4, 419-428 (1999).

- [5] Hermel, D., Hasheminia, H., Adler, N. and Fry, M. J., “A solution framework for the multi-mode resource-constrained cross-dock scheduling problem,” *Omega*, 59, 1, 157-170 (2016).
- [6] Konur, D., and Golias, M. M., “Cost-stable truck scheduling at a cross-dock facility with unknown truck arrivals: A meta-heuristic approach,” *Transportation Research Part E: Logistics and Transportation Review*, 49, 1, 71-91 (2013).
- [7] Ladier, A. L. and Alpan, G., “Cross-docking operations: Current research versus industry practice”. Forthcoming in *Omega*.
- [8] Miao, Z., Lim, A. and Ma, H., “Truck dock assignment problem with operational time constraint within crossdocks,” *European Journal of Operational Research*, 192, 1, 105–115 (2009).
- [9] Soltani, R. and Sadjadi, S. J., “Scheduling trucks in cross-docking systems: A robust meta-heuristics approach,” *Transportation Research Part E: Logistics and Transportation Review*, 46, 5, 650–666 (2010).
- [10] Tsui, L. Y. and Chang, C. H., “A microcomputer based decision support tool for assigning dock doors in freight yards,” *Computers & Industrial Engineering*, 19, 1, 309–312 (1990).
- [11] Tsui, L. Y. and Chang, C. H., “An optimal solution to a dock door assignment problem,” *Computers & Industrial Engineering*, 23, 283–286 (1992).
- [12] Vahdani, B. and Zandieh, M., “Scheduling trucks in cross-docking systems: Robust meta-heuristics,” *Computers & Industrial Engineering*, 58, 1, 12–24 (2010).
- [13] Van Belle, J., Valckenaers, P. and Cattrysse, D., “Cross-docking: State of the art,” *Omega*, 40, 6, 827–846 (2012).
- [14] Van Belle, J., Valckenaers, P., Berghe, G. V. and Cattrysse, D., “A tabu search approach to the truck scheduling problem with multiple docks and time windows,” *Computers & Industrial Engineering*, 66, 4, 818–826 (2013).
- [15] Vis, I. F., and Roodbergen, K. J., “Positioning of goods in a cross-docking environment,” *Computers & Industrial Engineering*, 54, 3, 677-689 (2008).
- [16] Yu, W. and Egbelu, P. J., “Scheduling of inbound and outbound trucks in cross docking systems with temporary storage,” *European Journal of Operational Research*, 184, 1, 377–396 (2008).