THROUGHPUT RATE OF A TWO-WORKER STOCHASTIC BUCKET BRIGADE

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Abstract

Work-sharing in production systems is a modern approach that improves throughput rate. Work is shifted between cross-trained workers in order to better balance the material flow in the system. When a serial system is concerned, a common work-sharing approach is the Bucket-Brigade (BB), by which downstream workers sequentially take over items from adjacent upstream workers. When the workers are located from slowest-to-fastest and their speeds are deterministic, it is known that the line does not suffer from blockage or starvation, and achieves the maximal theoretical throughput rate (TR). Very little is known in the literature on stochastic self-balancing systems with work-sharing, and on BB in particular. This paper studies the basic BB model of Bartholdi & Eisenstein (1996) under the assumption of stochastic worker speeds. We identify settings in which conclusions that emerge from deterministic analysis fail to hold when speeds are stochastic, in particular relating to worker order assignment as a function of the problem parameters.

1 Introduction

Mass production environments have gone through major changes due to the implementation of work-sharing. Traditionally, such systems were based on division of work among multiple workers, each trained to repeatedly perform a small segment of the work. Education and training improvements have provided incentives for workers to handle more complex requirements, thus providing the potential for enhancing system performance. Modern environments have applied work-sharing, whereby multiple cross-trained workers are able to perform the same task. The implementation of work-sharing has improved the ability to achieve balanced lines [1, 2], leading to fewer blockages and starvation in the system, and as a result, to a higher throughput rate (TR). Ostolaza et al. [3] were the first to coin the term Dynamic Line Balancing (DLB). This term refers to the operational side of work-sharing as "allowing tasks to be assigned on the fly based on the current state of the system". According to DLB, some of the tasks, defined as 'shared tasks' can be shifted between adjacent stations/workers, in order to better balance the flow along the line. Some rules were consequently applied to control the line dynamics.

One of the most common work-sharing approach is the Bucket-Brigade (BB), proposed by Bartholdi and Eisenstein [4]. This approach initially assumed full cross-training of the workers, and suggested conditions ensuring a self-balancing line. In the basic model, they also assumed that a task can be handed from one worker to another at any point, and that the system is deterministic in all respects. The basic rule of BB is that whenever the last downstream worker completes an item, the worker returns and takes over the item of the next upstream worker; the preempted worker continues similarly with the next upstream worker, where this procedure continues until the first worker is preempted and starts processing a new item. This procedure prevents starvation, however the authors show that when the workers are located from slowest-to-fastest, the line does not suffer from blockage as well, thus achieving the maximal theoretical throughput rate. Moreover, they show that in steady state the 'work taking over' between any two adjacent workers always occurs at the same point, meaning that each worker always executes exactly the same work segment on each item. Further analysis of the BB system dynamics for two and three workers was presented in [5], while the chaotic behavior of the hand-off point when the convergence condition does not hold was studied in [6]. For multiple extensions of BB, see [7].

Typical serial systems in which BB can be applied are production/assembly lines and order picking systems. In the latter, multiple pickers, located along an aisle, perform picking tasks from a flow rack (see Figure 1.1). The items of each order are picked into a box/tote by a picker, while moving toward the end of the aisle. When the last picker completes an order, she moves backward and takes over the box of the adjacent upstream picker, who does the same to the next upstream picker and so on.

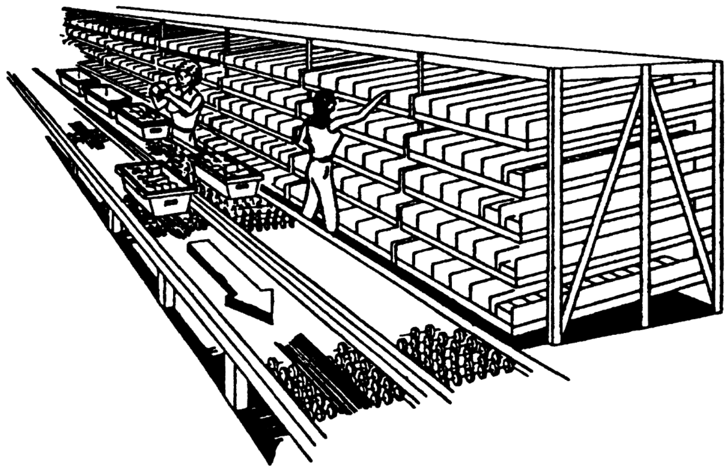

Figure 1.1: An order picking system (from Bartholdi et al. 2001)

Bartholdi et al. [8] relaxed the deterministic work content assumption, and examined the effectiveness of BB assuming stochastic work times under exponential distribution. They show analytically that, as the number of stations increases, the model converges to the optimal throughput when workers are assigned from slowest-to-fastest. Bratcu and Dolgui [9] studied stochastic speeds via simulations, while assuming that both working and walk back speeds are normally distributed. Hong [10] suggested a closed-form expression for the 2-worker blocking congestion in a circular-passage system, for constant workers' speed and stochastic pattern of picks. He showed via simulation that the analytical expression provides a good approximation for the blockage probability of a BB picking system, when the worker speed and hand-off time are constant and the picking pattern is stochastic.

As opposed to the knowledge on stochastic self-balancing systems without work-sharing, very little is known in the literature on such systems with work-sharing, and on BB in particular. In this paper we analyze the basic model of BB [4] under the assumption of stochastic worker speeds. We present the dynamics of the system, and provide analytic results to the case where one worker dominates the other. In the general case, when partial blockage may occur, we demonstrate the effect of the workers speed parameters on the TR, and show that in some cases, the fastest-to-slowest assignment in terms of expected speeds provides higher TR than the reverse order.

2 Model

Bucket Brigade (BB) is a decentralized dynamic protocol, which allows coordinating the efforts of several workers along a production line. Each worker moves down the line with an item. Passing downstream workers is not allowed, so each worker either proceeds at their own pace or at a reduced speed when blocked by a downstream worker. The last worker, upon completion of an item at the end of the line, returns to take over (hand-off) the item from the immediate upstream worker, who does the same, until the first worker takes a new item from the start of the line. We assume that the time required to return upstream is insignificant compared with the time required to work downstream.

2.1 Bucket Brigade dynamics

The dynamics of a two-worker system can be described with discrete events, where hand-off $n = 0, 1, 2, ...$, refers to the event by which the second worker has reached the end of the line completing an item, and returns to receive the item from the first worker. A state of the system, x_n, defines the position along the line of hand-off n, i.e. the position of worker 1 when worker 2 has reached the end of the line. Without loss of generality, we assume that the length of the line is 1, so that $0 \leq x_n \leq 1$. At each hand-off, the worker speeds, v_i, $i = 1, 2$, are randomly and independently generated according to known, stationary cumulative distribution functions (cdf) $F_i(\xi)$, $\xi \in [0, \infty)$.

In case worker 1 is blocked by worker 2, the two workers reach the end of the line concurrently, and the hand-off takes place at the position $x_n = 1$. Following the BB rule, the next hand-off takes place immediately and with certainty at the position $x_n = 0$. Since worker 2 cannot be blocked, and always reaches the end of the line, the time elapsed between hand-off n and hand-off $n + 1$ is always $\frac{1-x_n}{v_2}$. Thus, the position of hand-off $n + 1$ is calculated recursively by,

$$x_{n+1} = \min\left\{1,\ \frac{v_1}{v_2}(1 - x_n)\right\}, \quad \text{where } x_0 \text{ is given.} \tag{2.1}$$

Analyzing this stochastic system as a Markov chain, we are interested in determining the steady state distribution of hand-off positions along the line, the expected cycle time, and the throughput rate. In steady state, the hand-off position, x, satisfies

$$x = \min\left\{1,\ \frac{v_1}{v_2}(1-x)\right\}, \tag{2.2}$$

which is obtained from (2.1) by omitting the hand-off index.

2.2 Throughput rate

As discussed above, the time elapsed between consecutive hand-offs in a BB production line is $\frac{1-x}{v_2}$, thus the expected cycle time of the line is

$$E\left[CT\right] = E\left[\frac{1-x}{v_2}\right] = \left(1 - E[x]\right) E\left[\frac{1}{v_2}\right], \tag{2.3}$$

where the second equality follows because v_2 is drawn independently from the hand-off position, x. The TR of the BB line when the workers are assigned in the order $1 \to 2$, is denoted by $TR^{1\to 2}$, and is equal to the inverse of the cycle time,

$$TR^{1\to 2} = \left(1 - E[x]\right)^{-1} E\left[\frac{1}{v_2}\right]^{-1}. \tag{2.4}$$

From 2.3 we can see that the TR of a single worker i, is equal to

$$TR_i = E\left[\frac{1}{v_i}\right]^{-1}. \tag{2.5}$$

The TR of a BB line, as shown in (2.4), depends on TR_2, i.e., $E\left[\frac{1}{v_2}\right]^{-1}$, and on the expected hand-off position, $E[x]$. Thus $TR^{1\to 2}$ combines the efforts of the two workers. Define $TR_i^{1\to 2}$ for $i = 1, 2$ as the TR of worker i when working in a BB line consisting of workers 1 and 2 in this order. The next proposition shows that the TR of worker 1 in a BB line is bounded from above by this worker's expected speed, and the TR of worker 2 equals the corresponding single worker TR, i.e., does not depend on the distributions of v_1 and x.

Proposition 2.1. *In a BB line,*

$$TR_1^{1\to 2} \leq E[v_1]$$

and

$$TR_2^{1\to 2} = TR_2.$$

Proof. All proofs are given in [11] $\square$

Contrary to worker 2, the TR of worker 1 depends on all parameters of the BB line. It is obtained by subtracting $TR_2^{1\to 2}$ from the TR of the line,

$$TR_1^{1\to 2} = TR^{1\to 2} - TR_2^{1\to 2} = E\left[\frac{1}{v_2}\right]^{-1} \frac{1}{1-E[x]} - E\left[\frac{1}{v_2}\right]^{-1} = E\left[\frac{1}{v_2}\right]^{-1} \frac{E[x]}{1-E[x]}.$$

3 Throughput rate in the case of dominance

In an instance of a BB line with stochastic speeds, worker 1 may be blocked by worker 2 in some iterations, resulting in subsequent hand-offs at $x_n = 1$ and $x_{n+1} = 0$. In other iterations, where worker 1 is not blocked, the hand-offs occur in some $0 < x_n < 1$. This section deals with particular cases of no blockage, where worker 1 is never blocked, and full blockage, where worker 1 is blocked at each hand-off. These cases come up when one of the workers in the team is almost surely faster than the other, i.e. with probability 1 has a higher or equal speed at each hand-off.

Assume, without loss of generality, that worker 2 dominates worker 1, i.e., $v_1 \leq v_2$ for sure. The two assignments, $1 \to 2$ and $2 \to 1$, correspond to the slowest-to-fastest and fastest-to-slowest rules, respectively, which are well-defined only in the case of dominance. Denote $TR^{i\to j}$ as the throughput rate of a 2-worker BB line with aorkers i and j in that order, and $TR_k^{i\to j}$, as the throughput rate of worker $k = i, j$ in this line. The slowest-to-fastest personnel assignment rule is known to be optimal in terms of TR when workers have deterministic speeds [4]. The rule allows avoiding blockage and attains the maximum TR,

$$TR^{1\to 2} = v_1 + v_2. \tag{3.1}$$

The opposite, fastest-to-slowest assignment leads to full blockage in the deterministic environment, with the TR, $TR^{2\to 1} = 2v_1$, slower than that in (3.1).

Under stochastic speeds, the question of setting an appropriate personnel assignment rule is significantly more complicated. However, in the case of dominance, the slowest-to-fastest assignment is better than the opposite one, as proved in the next proposition. The proposition also explicitly calculates the TR of the workers and of the line.

Proposition 3.1. *If worker 2 dominates worker 1, then*

- *the slowest-to-fastest assignment yields*

$$TR_1^{1\to 2} = E\left[v_1\right] \leq TR_2^{1\to 2}, \quad TR^{1\to 2} = E\left[v_1\right] + E\left[\frac{1}{v_2}\right]^{-1}, \tag{3.2}$$

- *the fastest-to-slowest assignment yields*

$$TR_2^{2\to 1} = TR_1^{2\to 1} = E\left[\frac{1}{v_1}\right]^{-1}, \quad TR^{2\to 1} = 2E\left[\frac{1}{v_1}\right]^{-1}, \tag{3.3}$$

- *the slowest-to-fastest assignment is better:* $TR^{1\to 2} \geq TR^{2\to 1}$.

Proposition 3.1 shows that under dominance, the TR of the first worker can never exceed that of the second worker. Additionally, while the TR in BB depends on the whole distribution of the speed of worker 2, it depends on the speed of worker 1 only through the expected value.

4 General Case: Numerical Approximation

In this section we analyze numerically (using Wolfram Mathematica) an approximate solution to cases where partial blockage may exist, and gain some additional insights on the stochastic behavior of BB. The speeds of the two workers are taken from Beta distribution. As an illustration, Figure 4.1 depicts the TR as a function of the expected worker speeds e_i and their standard deviations s_i. This figure demonstrates how the conclusions of Proposition 3.1 extend to the case of partial blockage, showing that the TR increases with both workers' expected speed and decreases with their standard deviation. The effect of both parameters on the TR is significantly more substantial for the second worker as compared to the first worker. In particular, the TR is close to zero when the coefficient of variation of the second worker's speed is close to 1. Since the standard deviation of the second worker has significant effect on the TR of the line, the slowest-to-fastest order based on the expected speeds is not always optimal. In particular, if both workers have the same expected speed, it is better to located the one with the larger standard deviation first. However, when there is a relatively large difference between the standard deviation of the two workers, it is often optimal to locate the worker with the highest expected speed first, namely, fastest-to-slowest order based on the expected speed.

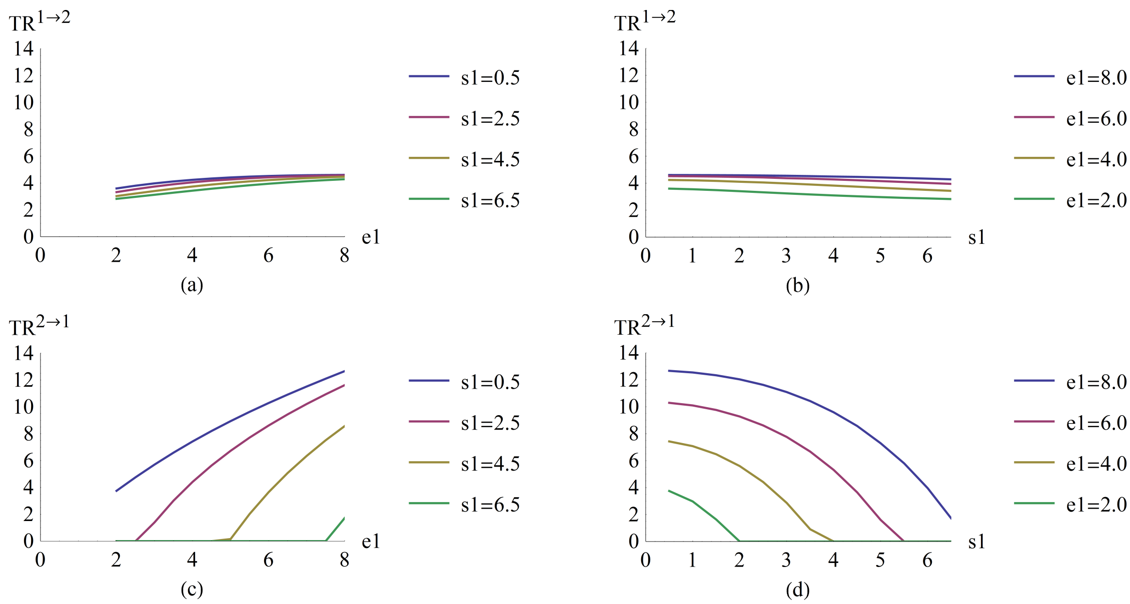

Figure 4.1: Throughput rate ($e_2 = 5$, $s_2 = 1$)

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