TOWARD SUSTAINABILITY, HIGH DENSITY AND SHORT RESPONSE TIME BY LIVE-CUBE STORAGE SYSTEMS

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Abstract

This paper studies random storage in a live-cube storage system where loads are stored multi-deep. Although such storage systems are still rare, they are increasingly used, for example in automated car parking systems. Each load is accessible individually and can be moved to a lift on every level of the system in x- and y-directions by a shuttle as long as an open slot is available next to it, comparable to Sam Loyd's sliding puzzles. A lift moves the loads across different levels in z-direction. We derive the expected travel time of a random load from its storage location to the input/output point. We optimize system dimensions by minimizing the expected travel time.

1 Introduction

Live-cube storage systems are recently introduced automated storage systems which can achieve high storage density together with short response times. In live-cube storage systems, the highest storage density can be achieved while unit loads can individually move in a 3-dimensional space. They have many applications, which can be found in parking systems (e.g. "Park, Swipe, Leave" parking systems [1], "Space Parking Optimization Technology" or SPOT [2], "Hyundai Integrated Parking" or HIP Systems [3], "Wohr Parksafe" [4]). They are also applied in warehouses and cross-dock systems (e.g. "Magic Black Box" [5]) and container yards (e.g. "Ultra-high Container Warehouse" or UCW systems [6]).

Such storage systems operate with electrically powered shuttles and lifts, which lead to significantly reduced fossil fuel and energy consumption, and CO2 emissions. Table 1 compares the energy consumption and CO2 emission of a typical live-cube and a traditional multi-storey car parking system of the same capacity (192 cars), and for different types of power plants generating the energy needed for operation (lighting, ventilation, moving the cars).

Table 1. Energy consumption and CO_2 emissions of live-cube parking system and traditional multi-storey car park*

Generated by	Fossil-fuel power plant		Nuclear power plant		Biomass-fuel power plant	
Parking type	Live-cube parking	Traditional car park	Live-cube parking	Traditional car park	Live-cube parking	Traditional car park
Average CO_2 emission (gram/car)	96	4369	1	200	0	184
Average S/R energy consumption (kWh/car)	0.12	4.94	0.12	4.94	0.12	4.94
Average lighting energy consumption (kWh/car)	0.00	0.25	0.00	0.25	0.00	0.25
Average ventilation energy consumption (kWh/car)	0.00	5.00	0.00	5.00	0.00	5.00
Total (kWh/car)	0.12	10.19	0.12	10.19	0.12	10.19

*Input data retrieved from [3] and [7].

As Table 1 shows, a live-cube parking system significantly reduces the energy consumption and CO_2 emissions compared to the traditional multi-storey car park. This saving is even more significant for CO_2 emissions if electricity is provided by a fossil-fuel power plant.

A live-cube storage system contains multiple levels of storage grids, shuttles, a lift, and a depot, or an Input/Output (I/O) point. Shuttles can move in x- and y- directions (as long as there is an empty space) while carrying a unit load. These moving patterns can be compared to solving a Sam Loyd's 15-puzzle game [8]. A lift takes care of movements across different levels in z-direction (see Figure 1). We assume the I/O point is located at the lower left corner of the system. When idle, the lift waits at the I/O point. The performance of a storage system in service industries is often measured in terms of its response time. This paper optimizes dimensions of a live-cube storage system under random storage policy. In order to do this, we define a mathematical model for the expected retrieval time of an arbitrary unit load as a function of system dimension sizes.

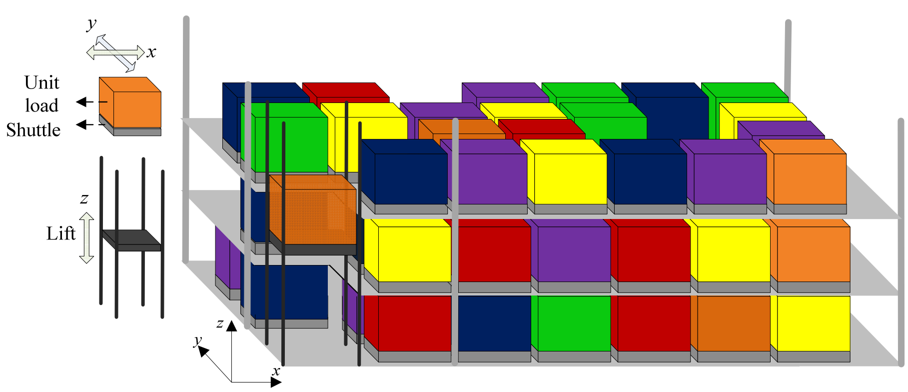

Figure 1. A live-cube storage system

2 Mathematical model

A random retrieval location can be denoted by (X ,Y ,Z) where X, Y and Z refer to coordinates in x-, y- and z- directions respectively. The system capacity is a known positive constant. A random storage policy is assumed. It is also assumed that the utilization of the system cannot exceed $(V' - \max\{L,W\})/V'$, where V', L, and W represent the capacity of the system in number of storage locations, the number of columns in each level, and the number of rows in each level, respectively.

Theorem 1. *If there is at least one empty location in each row and each column of each level of a live-cube storage system (i.e. max utilization* $\le (V' - \max\{L,W\})/V'$*), the minimum retrieval time of a random unit load stored at location* (X, Y, Z)*, can be estimated by the following equation:*

$$T(X,Y,Z) = \max\{X+Y, Z\} + Z\ . \tag{1}$$

Proof. Theorem 1 can be proven by using mathematical induction, which is omitted here.

Using this theorem, we obtain the expected retrieval time given by Equation (2) and the mathematical model of the problem as below (Model MGM):

$$\min \sum_{i\in\{A,B,C,D\}} u_i E[T_i]\ , \tag{2}$$

subject to:

$$lwh = V\ , \tag{3}$$

$$l - w \ge 0\ , \tag{4}$$

$$\sum_{i\in\{A,B,C,D\}} u_i = 1\ , \tag{5}$$

$$u_A(w-h) \ge 0\ , \tag{6}$$

$$u_B(h-w) \ge 0\ , \tag{7}$$

$$u_B(l-h) \ge 0\ , \tag{8}$$

$$u_C(h-l) \ge 0\ , \tag{9}$$

$$u_C(l+w-h) \ge 0\ , \tag{10}$$

$$u_D(h-l-w) \ge 0\ , \tag{11}$$

Decision variables: $l > 0, w > 0, h > 0$,

and $u_i \in \{0,1\}$ for $i \in \{A,B,C,D\}$.

Equation (2) minimizes the expected retrieval time $E[T]$. Constraint (3) makes sure that the given capacity (V) is achieved. Constraint (4) ensures the length is at least equal to the width of the system. Constraint (5) guarantees exactly one of the cases is considered in the objective function. Constraints (6)-(11) take care of the feasibility of the solutions of each case. Length (l), width (w) and height (h) are expressed in time units.

The model is non-linear and mixed integer; however, we can optimally solve it by splitting it into several solvable sub-models and reducing the feasible area of the decision variables without losing the optimal solution. In order to solve the model we have to derive the expected retrieval time in Equation (2). The expected retrieval time for any live-cube system with a given capacity can be calculated as follows:

$$E[T] = \int_{t=0}^{\max\{w+l,h\}+h} tf(t)dt \,, \tag{12}$$

where, t represents the retrieval time for any retrieval location. $f(t)$ represents the probability density function of retrieval time $t, 0 \le t \le \max\{w+l,h\}+h$. In order to calculate the expected retrieval time, we need to derive $f(t)$. By knowing the cumulative distribution function of the retrieval time ($F(t)$) we can then derive $f(t)$. The cumulative distribution function can be calculated as follows:

$$F(t) = P(T \le t) = P(\max\{X+Y, Z\} + Z \le t) = P(X+Y+Z \le t \cap 2Z \le t)\,. \tag{13}$$

The two conditions, $X+Y+Z \le t$ and $2Z \le t$ are not independent of each other and therefore cannot be separated. Figure 2(a) illustrates the optimal shape which includes all the locations with retrieval time less than or equal to *t*. Therefore, for any value of retrieval time, *t*, the probability that the random variable *T* is less than or equal to *t* can be calculated as:

$$F(t) = P(T \le t) = \frac{\text{volume of the region } T \le t \text{ in the system}}{\text{volume of the system}} \tag{14}$$

(a) **(b)**

Figure 2. (a) The region formed by $x+y+z \le t$ and $2z \le t$ (b) cubic shape of a multiple-level system with its corner points

However, the region in Figure 2(a) may be restricted by the cubic system. The cubic shape of multiple-level live-cube system is illustrated in Figure 2(b). Therefore, it may not possible to include all the locations with the retrieval time less than or equal to *t* because of the system restriction. The shape in Figure 2(a) will be transformed to different shapes depending on relative sizes of rack dimensions (system configuration) and retrieval time *t*. Each shape is related to a specific formula, which returns the volume of the shape and therefore we can derive for each shape a specific cumulative and probability density function. The classification is due to different ways of calculating the probability density function in each case other than the other cases. Each case can be shown to have only one formula for $E[T]$.

By simultaneously considering two conditions, the calculation of $E[T]$ can be done into four different complementary cases, each referring to a specific configuration of the system. The four cases of system configuration are listed as follows:

- case A: $h \le w$,
- case B: $w < h \le l$,
- case C: $l < h \le l + w$,
- case D: $l + w < h$.

3 Results

We obtain the optimal solution of Model MGM by comparing the solutions of four cases. The following equations give optimal values of $E[T_A]$, $E[T_B]$, $E[T_C]$, and $E[T_D]$ as a function of volume of the system V.

$$E[RT_A^*] = 1.53097 V^{1/3} \tag{15}$$

$$E[RT_B^*] = 1.53789 V^{1/3} \tag{16}$$

$$E[RT_C^*] = 1.54167 V^{1/3} \tag{17}$$

$$E[RT_D^*] = 1.81889 V^{1/3} \tag{18}$$

As it can be seen from Equations (15-18), the solution of case A ($h \le w$) gives the minimal $E[T]$ for Model MGM. Figure 3 illustrates the optimal $E[T]$ of four cases for varying volume of the system.

Therefore, the Equations (19) and (20) give the optimal solutions of Model MGM. For any volume of the system, a system with the following dimension sizes is the system with minimum expected retrieval time.

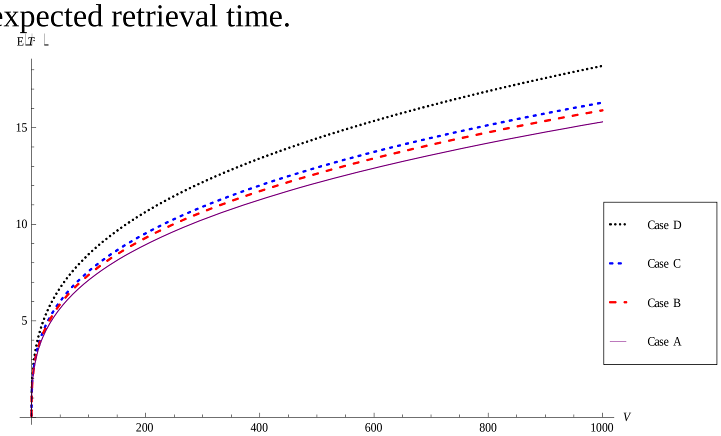

Figure 3. Optimal $E[T]$ for cases A, B, C, and D versus system volume

$$h^*(V) = 0.874461V^{1/3} \tag{19}$$

$$w^*(V) = l^*(V) = 1.069374V^{1/3} \tag{20}$$

4 Conclusion

A live-cube system can realize high storage density since virtually no transportation aisles are needed. In addition, the system significantly reduces the energy consumption needed for operation and CO_2 emissions compared to a traditional storage system (as shown in our car park example). The system can respond fast to customer orders due to independent and simultaneous movements of its components in 3-dimensional space. One of the most important performance measures is the customer response time. In this study, we derive the expected retrieval time of the system as a measure to compare the performance of such a system with other storage systems under random storage policy. However, the response time of such a system is heavily dependent on its configuration. Therefore, we propose a mathematical model to obtain the optimal dimensions of the system leading to the minimum response time. The model can be optimally solved by splitting it into several solvable sub-models without losing the optimal solution.

Several research questions regarding the live-cube storage systems remain open. While we have studied live-cube storage systems with lifts, results for other live-cube storage systems with different vertical movement mechanism may also prove worthwhile investigating. It is also possible to study the live-cube storage system with other storage policies such as class-based storage policy and compare the results with the results obtained here in this study.

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