# MODELING THE INVENTORY REQUIREMENT AND THROUGHPUT PERFORMANCE OF PICKING MACHINE ORDER-FULFILLMENT TECHNOLOGY

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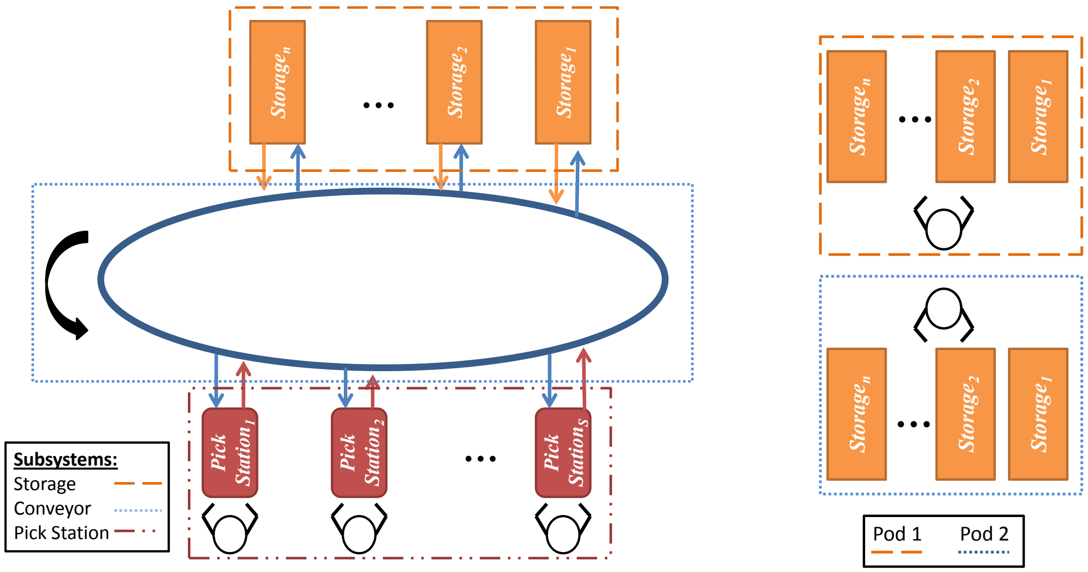
**Abstract**

Picking machines, also known as remote-order-picking systems, are an example of a stock-to-picker piece-level order-fulfillment technology that consists of two or more pick stations and a common storage area. An integrated closed-loop conveyor decouples the pick stations from the storage area by transporting the needed totes to and from the storage area and the pick stations. We develop a probabilistic model capable of quantifying the inventory differences between order-fulfillment technologies that pool inventory with technologies that do not pool inventory. To determine the throughput of a picking machine, we develop a methodology that incorporates existing analytical models for the picking machine's subsystems. We present a case study comparing a picking machine to a carousel-pod system to illustrate how a manager could use our methodology to answer system design questions. Finally, we present conclusions and future research.

## 1. Introduction

Order-fulfillment, the process of fulfilling customer demand through the transfer of a set of items from inventory, is the most critical task in the distribution process of an organization because of its simultaneous impact on the cost and the accuracy of the process [5]. Order-fulfillment requires the application of resources such as inventory, labor, and information; therefore, designing an effective order-fulfillment process is an important aspect in distribution center design. In this paper we consider piece-level fulfillment as opposed to unit-load or carton-based fulfillment.

There are three typical piece-level fulfillment strategies: picker-to-stock, stock-to-picker, and utilizing an automated dispensing system. In a picker-to-stock strategy, an operator visits fixed locations to make a pick. In a stock-to-picker strategy, the items to be picked are




(a) A Picking Machine (b) A Carousel-Pod System

Figure 1: Aerial Views of a Picking Machine and a Carousel-Pod System

transported to the operator; example technologies include carousels, vertical lift modules (VLMs), mini-load automated storage and retrieval systems (AS/RSs), and picking machines. Finally, in an automated dispensing strategy, automated order-picking machines are used to completely eliminate manual picking.

To meet customer requirements or to increase worker productivity, technology is often employed in the order-fulfillment process. A picking machine, also known as a remote-order-picking system or an automated storage and order fulfillment system [2, 16], is illustrated in Figure 1(a). A picking machine consists of numerous pick stations that access a common inventory storage area, which is usually multiple carousel units or a mini-load AS/RS where stock keeping units (SKUs) are stored in totes. An integrated conveyor transports the requested totes to and from the storage area and the pick stations; therefore, even though a picking machine has numerous pick stations, all pick stations utilize the same storage area. The required orders' shipping containers are transported via a conveyor to a pick station. At the same time, the requested SKUs' totes are retrieved automatically from the storage area and are also sent to the pick station. At the pick station, pick lights indicate to the operator both the position and the item quantity to be picked and put lights direct the operator to the position of the container to which the items are to be transferred (the put operation).

High-value products and short lead time requirements cause picking machines to be used in pharmaceutical, mail order, electronic, and retail distribution facilities. Picking machines can reduce labor costs by eliminating the walking and searching associated with picker-to-stock systems and can provide high-product density by using vertical space effectively. Depending on the design configuration, an order-fulfillment rate of up to 1,000 order lines per person-hour is possible [16].

Horizontal carousels are also a stock-to-picker piece-level technology. Carousels are commonly utilized in pods with more than one carousel unit per pod, as shown in Figure 1(b). This configuration allows one carousel in the pod to rotate to the next pick location while the operator retrieves an item from another carousel in the pod. Pick lights inform the operator of the position and item quantity to be picked. After picking the items and putting them in the correct tote, the operator walks to another carousel in the pod and picks again. The rotation time of a carousel is a function of the horizontal length of the carousel; consequently, the number of carousels in a pod is frequently configured such that the operator is the system throughput bottleneck.

As carousel systems are typically limited by throughput capacity (rather than by space capacity) [7], multiple carousel pods may be implemented to meet throughput requirements. One operator is assigned to each carousel pod and batch picking is usually performed. Typically all the items for a batch of orders are stored in each carousel pod; consequently, SKUs can be stored in multiple locations within the carousel order-fulfillment area. Pazour [11] discusses the relative advantages of picking machines over stand alone carousel systems.

The broad goal of our research is to systematically analyze alternative order-fulfillment technologies, comparing inventory and throughput requirements of technologies that pool inventory with technologies that do not pool inventory. In our work we focus on comparisons with carousel systems. Picking machines can be installed with a storage area that is either a carousel system or a mini-load AS/RS. For comparison purposes, in this paper we only consider picking machines that are serviced via a carousel system; however, our analyses could be extended to consider other stock-to-picker systems.

Our contribution lies in developing a methodology that incorporates multiple analytical models to determine whether to implement a picking machine or a carousel-pod system in a distribution center. As depicted in Figure 1(a), a picking machine can be divided into three subsystems: a storage subsystem, a pick station subsystem, and a closed-loop conveyor subsystem. To model a picking machine's inventory requirements and throughput performance, we first model each of the subsystems independently. We analyze the inventory requirements of the storage subsystem by developing a probabilistic model for the inventory differences between a picking machine and carousel-pod system. Because the subsystems are connected, an upper bound on the throughput of a picking machine is the minimum of the three subsystems' throughputs. We determine the throughput by applying and combining analytical models for each of the subsystems. We provide a case study comparing a picking machine to a carousel-pod system to illustrate how our picking machine subsystem models can be combined to answer design questions. Finally, we conclude the paper and discuss opportunities for future research.

## 2. Literature Review

Whereas a vast array of academic research exists on a picking machine's subsystems, we are aware of only a few articles that directly address picking machines.

The early literature includes two conference papers with preliminary simulation results that address picking machines. Perry et al. [14] use a discrete-event simulation model to assist in the physical system design of a picking machine by determining the number and dimension of storage aisles, number of work stations, conveyor geometry, number of inventory buffer positions. A simple, expected-value model is used to determine initial values for these design parameters, which are then modified based on simulation results and throughput requirements. Their testing indicates that the conveyor subsystem is the bottleneck on system throughput. Raghunath et al. [15] describe an interactive and flexible simulation structure for a picking machine.

Park [9] determines the first and second moments of the cycle time of a carousel system that performs sequential retrievals for a picking machine. By assuming the retrieval requests occur according to a Poisson distribution, Park also determines the expected waiting time of a retrieval request at the carousel system.

Picking machines have been mentioned in the literature as future research by Bozer and White [2]: "A possibility is to investigate the use of remote picking stations where each station is interfaced to the storage/retrieval system via a closed conveyor loop. Such a system allows each picking station access to the aisles." Also, Park et al. [10] classify mini-load AS/RSs into three categories, one mentioned as future research is a closed-loop conveyor: "mini-load systems containing a closed-loop conveyor, often called the remote order picking system, have a closed-loop conveyor system to deliver the containers that interconnects each aisle of the mini-load system with the remote order picking stations."

## 3. Probabilistic Inventory Difference Model

An advantage of a picking machine over a carousel-pod system is that numerous pick stations can utilize the same storage locations, as the conveyor subsystem decouples the pick stations from the storage area. To realize the inventory advantages of a common storage area, an infrastructure investment in a closed-loop conveyor subsystem is required.

Ideally, distribution centers want to fulfill each order (in its entirety) at a single carousel pod. To ensure this, we assume each carousel pod will store the same assortment of SKUs, requiring a SKU to be stored in multiple locations. We revisit this assumption in more detail in Section 3.1.1. A facilities engineer faced with purchasing either a picking machine or a carousel-pod system would need to answer the following question: What amount of inventory savings will need to be created by the picking machine system to compensate for the additional costs of an integrated conveyor? To answer this question, we create an analytical model to estimate the inventory differences between the two systems. First, we introduce the following notation.

**Sets:**

$S$ SKUs; indexed by $s = 1, 2, \ldots, |S|$

**Parameters:**

$p$ number of pods

**Variables:**

$d_s^p$ number of totes for SKU $s$ in a $p$-pod system
$N(p)$ total totes in a $p$-pod system

A picking machine has a single storage area and thus has inventory requirements equivalent to a 1-pod carousel system. In a picking machine, inventory levels are dictated by demand characteristics of each SKU. The number of totes for a SKU in a picking machine is the required number of items of inventory divided by the number of items that can fit in a tote, rounded up to an integer value. The number of totes for a SKU in a carousel system with $p$ pods is the maximum of the number of totes in a 1-pod system and the number of pods. The total number of totes, $N(p)$, is the sum of the number of totes for each SKU in the system.

If the number of totes required in a 1-pod system is not divisible by the number of pods, we assume the extra totes are randomly allocated amongst the carousel pods. Therefore, the number of totes for each SKU does not have to be the same for every pod and the number of totes in each pod does not have to be equal (albeit, the number of totes in each carousel will be very similar).

To compare the inventory requirements in a picking machine and a carousel pod, we present the following example. Assume we have 12 SKUs as shown in Table 1 with given inventory requirements. Also, assume for simplicity that each SKU has 50 items per tote. Table 1 provides the number of totes for a 1-pod system (a picking machine) and for a 2-, 3-, and 4-pod system. As the number of pods increases, the total number of totes in the system also increases. When $p \geq 4$, all SKUs will require $p$ totes because each SKU in these data requires at most 4 totes in a 1-pod system (i.e., when $p \geq 4$, $d_s^1 \leq 4\ \forall s \in S$ or $max\{d_s^1, p\} = p\ \forall s \in S$).

When the maximum number of totes required by a SKU in a picking machine is greater than or equal to the number of pods, the total totes in a $p$-pod system can be estimated as a linear function of $p$, as shown in Theorem 1.

**Theorem 1**

$$\textit{If } \max_{s \in S}\{d_s^1\} \leq p, \textit{ then } N(p) = p|S|.$$

**Proof:** $d_s^1 \leq p \quad \forall s \in S$ (because $\max_{s \in S}\{d_s^1\} \leq p$). Therefore, $d_s^p = p \quad \forall s \in S$.
So $N(p) = \sum_{s \in S} d_s^p = \sum_{s \in S} p = p|S|$. $\square$

Table 1: An Example Illustrating the Inventory Requirements of Various $p$-Pod Systems

| | | Number of totes ($d_s^p$) | | | |
|---|---|---|---|---|---|
| $s$ | *inv.* | $p=1$ | $p=2$ | $p=3$ | $p=4$ |
| A | 175 | 4 | 4 | 4 | 4 |
| B | 150 | 3 | 3 | 3 | 4 |
| C | 100 | 2 | 2 | 3 | 4 |
| D | 75 | 2 | 2 | 3 | 4 |
| E | 50 | 1 | 2 | 3 | 4 |
| F | 25 | 1 | 2 | 3 | 4 |
| G | 15 | 1 | 2 | 3 | 4 |
| H | 10 | 1 | 2 | 3 | 4 |
| I | 1 | 1 | 2 | 3 | 4 |
| J | 1 | 1 | 2 | 3 | 4 |
| K | 1 | 1 | 2 | 3 | 4 |
| L | 1 | 1 | 2 | 3 | 4 |
| $N(p)$ | | 19 | 27 | 37 | 48 |

To demonstrate how Theorem 1 can be used as an upper bound on the percent increase in inventory, we simulate different demand curves. We represent the ABC curve as presented by Bender [1], where the $x/y$ curve indicates that $x\%$ of the SKUs make up $y\%$ of the total demand. Figure 2 illustrates that as the number of pods increases, so does the required total number of totes in the system. Additionally, as demand skewness decreases, the increase in inventory approaches a linear function (in fact, the inventory increase for the 20/21 curve is a linear function). Linear trend lines are fit to each demand curve's inventory increase. The slope of the fitted line ($b$) and the associated $r^2$ values are shown in Figure 2. All fitted lines achieve an $r^2$ value greater than or equal to 0.9987, indicating that the percent increase in inventory has an approximately linear relationship with the number of pods. Additionally, all fitted lines have a slope that is greater than or equal to 0.72, illustrating that the additional inventory requirements of a carousel system are significant. This is especially important when storing high-valued products, which is a primary market for picking machine technology.

### 3.1 An Expected Value Model for the Number of Totes in a $p$-Pod Carousel System

Next, we develop an expression for the expected value of the number of totes in a carousel system with $p$ pods. This expected value is based on the concept of a SKU being either *low* or *high*. We define a SKU as *low* if the number of copies of a SKU in a picking machine is less than or equal to the number of pods in the carousel system (i.e., if $d_s^p \leq p$). Additionally, we define a SKU as *high* if the number of copies of a SKU in a picking machine is greater than the number of pods in the carousel system (i.e., if $d_s^p > p$). Note,

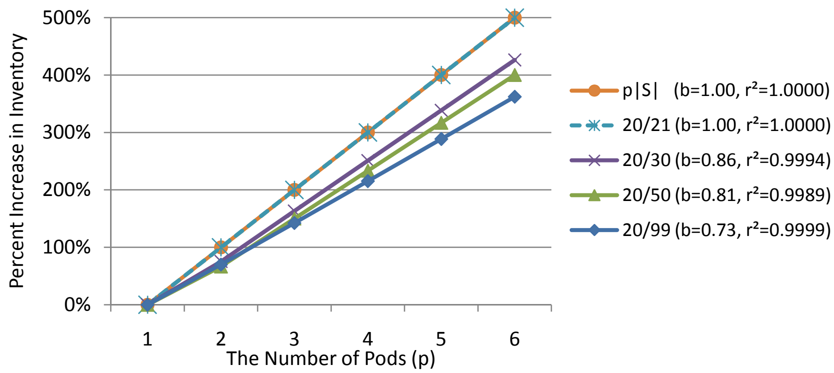



Figure 2: The Impact of Demand Skewness on Inventory Increases

the classification of a SKU as low or high is relative and depends on the number of carousel pods; thus, a SKU's classification varies as $p$ varies. No inventory benefit is associated with a high SKU in a picking machine. Instead, a low SKU generates inventory savings in a picking machine because carousel systems require duplicate copies of this SKU. By conditioning on whether a SKU is a low SKU or a high SKU, we create an expected value model for the number of totes in a $p$-pod system ($E[N(p)]$), provided in (1).

Let $f(z)$ be the probability that a SKU has $z$ totes in a picking machine and $F(z)$ denote the cumulative distribution of $z$, where $F(z) = \sum_{0}^{z} f(z)$. Then,

$$\begin{aligned} E[N(p)] &= E[N(p)|z \leq p]Pr(z \leq p) + E[N(p)|z > p]Pr(z > p) \\ &= |S|(p \times F(p) + E[z|z > p] \times (1 - F(p))), \end{aligned} \tag{1}$$

where

$$E[z|z > p] = \sum_{p}^{m} \frac{zf(z)}{c}, \qquad c = 1 - F(p), \text{ and } m = \max_{s}\{d_s^1\}.$$

In (1), a low SKU receives $p$ copies and the probability that a SKU is a low SKU is the $Pr(z \leq p)$ or $F(p)$. A high SKU receives $z$ copies and the probability that a SKU is a high SKU is the $Pr(z > p)$ or $1 - F(p)$. We apply our analytical model to an example with 2,000 SKUs and assume that each SKU follows the following distribution for the number of totes per SKU in a picking machine: $f(z) = 0.65$ for $z = 1, 0.20$ for $z = 2, 0.09$ for $z = 3, 0.03$ for $z = 4, 0.02$ for $z = 5$, and 0.01 for $z = 6 \quad \forall s \in S$.

Table 2 provides results from our 2,000-SKU example. The total expected number of totes that are a low or high SKU are denoted as $E[N(p)|z \leq p]$ and $E[N(p)|z > p]$, respectively. As the number of pods increases, so does the expected total number of totes in the system. This occurs because more SKUs are classified as a low SKU and less SKUs are classified as a high SKU. For example when $p = 1$, 65% of SKUs are considered a low SKU, resulting in 1,300 totes ($2,000 \times 1 \times 0.65$). The remaining 35% of SKUs are classified as a high SKU, resulting in 1,900 totes ($2,000 \times 2.71 \times 0.35$). For $p = 6$, all

SKUs are classified as a low SKU (i.e., none are a high SKU); consequently, each SKU receives six copies regardless of its demand profile, resulting in 12,000 total totes and all are classified as a low SKU ($2,000 \times 6 \times 1.0$).

Table 2: Results from our Example

| $p$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F(p)$ | 0.65 | 0.85 | 0.94 | 0.97 | 0.99 | 1.00 |
| $E[z \mid z > p]$ | 2.71 | 3.67 | 4.67 | 5.33 | 6.00 | – |
| $E[N(p) \mid z \leq p]$ | 1,300 | 3,400 | 5,640 | 7,760 | 9,900 | 12,000 |
| $E[N(p) \mid z > p]$ | 1,900 | 1,100 | 560 | 320 | 120 | – |
| $E[N(p)]$ | 3,200 | 4,500 | 6,200 | 8,080 | 10,020 | 12,000 |

In our analysis, we use the increase in totes of inventory as a surrogate to represent the increase in inventory. We justify this representation because increasing the number of totes increases the holding cost associated with the inventory, both in the form of increased inventory holding costs and increased infrastructure capital costs.

We assume each SKU is to be stored in the picking machine and the carousel system, regardless of demand profiles. An alternative view is to create an assignment model where SKUs compete for inclusion in the picking machine (carousel system). If a SKU is not assigned to the picking machine (carousel system), the SKU would be picked conventionally. Accordingly, each SKU has a labor benefit associated with being assigned to the picking machine (carousel system) based primarily on the SKU's demand profile. Each SKU is assigned space in the picking machine (carousel system), so a benefit-to-space ratio for each SKU can be determined. In this scenario, a fewer number of SKUs would be allocated to a carousel system than to a picking machine because a low SKU in a carousel system would not generate a benefit-to-space ratio justifying an assignment to the carousel system.

This illustrates the potential of using picking machine technology for the order-fulfillment of slow-moving SKUs (the category with the greatest number of SKUs). In a carousel system, an additional fulfillment channel, such as a flow rack or pick-to-light system, customarily exists for slow-moving inventory. This additional channel is needed because the demand for the slow-moving SKUs is not high enough to warrant assigning one tote to each carousel pod. On the other hand, with a single storage location, slow-moving SKUs are assigned a single tote location per SKU; therefore, assignment of slow-moving SKUs in the picking machine can often times be justified. Consequently, only one channel of distribution (the picking machine) is needed in the order-fulfillment process, which has the potential to increase overall order-fulfillment throughput and customer responsiveness.

### 3.1.1 Generalization to the Expected Value Model

In practice, some SKUs may not be stored in every carousel pod due to their low-demand rates. To more accurately reflect this case, we adjust our expected value model by introducing a new parameter. Let $\alpha$ denote the percentage of low SKUs that are stored in only one carousel pod. Then (1) can be replaced with (2) to denote the expected value for the number of totes in a carousel system with $p$ pods:

$$E[N(p)] = |S|\{[(1-\alpha)p+\alpha] \times F(p) + E[z|z>p] \times (1-F(p))\}. \quad (2)$$

In the case when $\alpha \neq 0$, additional infrastructure and/or labor is required because every order cannot be fulfilled in its entirety at a single carousel pod. To accommodate these orders, carousel pods are connected either by a conveyor or by operators who manually walk orders from one pod to another. These configurations (which connect pick stations) are basically a carousel-pod system morphed into a picking machine.

## 4. Subsystem Throughput Model

We model the throughput performance of a picking machine by analyzing the storage, conveyor, and picking station subsystems independently. Because the subsystems are connected, an upper bound on the throughput of a picking machine is the minimum of the three subsystems throughputs. To determine the throughput of the storage subsystem, we apply an expected cycle-time model for a carousel with an S/R machine performing batch retrievals. We apply a stability-condition model to determine if the closed-loop conveyor subsystem will meet a given throughput requirement. Finally, we determine the number of pick stations required to meet a target throughput.

### 4.1 Storage Subsystem

To analyze the storage system subsystem, we require a cycle-time model for a carousel system that is serviced by an S/R machine. An expected single-command cycle time model for a carousel with an S/R machine performing batch retrievals is developed in [13] and we use this model to determine the expected cycle time of the storage subsystem. We provide the cycle-time model from [13] in Appendix A.

### 4.2 Conveyor Subsystem

A closed-loop conveyor subsystem integrates the storage and pick stations; consequently, for a picking machine to meet a throughput requirement, the conveyor subsystem also must meet this requirement. Bozer and Hsieh [3, 4, 6] analyze the performance of discrete-space, fixed-window, closed-loop conveyors with multiple loading and unloading stations by developing stability-condition models. A closed-loop conveyor can be analyzed by dividing the conveyor loop into segments and analyzing each segment independently [4], allowing us to analyze each subsystem of a picking machine independently.

Bozer and Hsieh's models assume Poisson-arrival rates to the conveyor and an infinite number of buffer spaces for totes waiting to be placed onto the conveyor. In [12], Pazour and Meller use a discrete-event simulation model of a picking machine to validate that the stability-condition model accurately predicts stability, even when key assumptions on

Poisson-arrival rates and infinite number of buffer spaces at conveyor loading stations are violated. Through testing, the authors show that five buffer spaces are adequate to achieve a given throughput performance [12] and so we assume this design configuration.

We use Bozer and Hsieh's model [4] to test if the closed-loop conveyor will be stable or unstable for a given conveyor velocity. This stability-condition model is provided in Appendix B. We illustrate how the stability-condition model can be applied in Section 5.

### 4.3 Pick Station Subsystem

The decoupling of the storage area from the pick process creates pick stations that operate independent of each other. Therefore, additional pick stations can be added to increase pick station throughput potential, with no impact on the storage subsystem. One operator is required at each pick station; consequently, determining the number of pick stations determines the labor requirements associated with a picking machine design.

The amount of time to process one tote at each pick station consists of the pick time and a portion of the batch setup time. Without loss of generality, we assume a constant pick time and batch setup time at each pick station, denoted as $W$ and $\psi$, respectively. For a given throughput requirement, the cumulative pick rate at each of the pick stations must be greater than the arrival rate of totes to the pick stations. Let $\theta$ denote the throughput requirement and $\Gamma$ the number of pick stations. The number of pick stations required to meet throughput can be calculated as,

$$\Gamma = \lceil \theta \times (W + \psi/n) \rceil. \tag{3}$$

In the next section we present a case study to illustrate how our inventory, carousel expected cycle-time, conveyor stability, and pick station models can be incorporated to guide strategic decisions.

## 5. Case Study: A Comparison of a Picking Machine and a Carousel-Pod System

In this section we illustrate how our methodology can be used to answer design questions. A strategic question for a distribution center manager is whether to implement a picking machine or a carousel-pod system. In such a design problem, the user would first need to specify a storage and a throughput constraint. Floorspace or clear height restrictions may also be considered. Through an example, we illustrate how to conduct an analysis that includes labor, infrastructure, and inventory requirements.

In this example, 2,500 SKUs are being considered for assignment to either a picking machine or a carousel-pod system. If a single storage area is used, 65% of the SKUs require 1 tote, 30% require 2 totes, and 5% require 3 totes. A throughput requirement of 1,500 totes (lines) per hour is desired. Operational characteristics for a picking machine and carousel-pod system are as follows: horizontal carousel speed equals 0.50 meters per second; vertical S/R machine speed equals 0.60 meters per second; carousel-pod's height equals 1.75 meters; picking machine carousel's height equals 4.50 meters; picking machine carousel's handoff time equals 4.00 seconds; start/stop time for carousel equals 2.00 seconds; and walking time between carousels equals 2.00 seconds (picking machine walk time equals 0 seconds). Carousel heights are chosen to minimize the number of S/R machines required. Each tote requires a storage dimension that is 0.50 meters long by 0.25 meters wide by 0.375 meters high. A conveyor window is 0.50 meters long and can handle one tote.

For comparison purposes, we assume the pick stations for a picking machine and a carousel-pod system are equivalent. For both technologies, we use a pick time of 3.00 seconds per tote, a batch size of 15 totes, and a setup time for a batch of 30.00 seconds. We believe this assumption is conservative because picking machine pick stations are fed by a conveyor such that totes arrive at a common point, which decreases the search time of the operator and simplifies the design of ergonomic pick stations.

To determine the floor space requirements of the storage area, $F$, we use the floor space calculation in [8]. This calculation is based on the number of pick faces, as well as a tote's length, $l$, and depth, $e$, requirements, as shown below,

$$F = \left(\frac{(m-2)l}{2} + 2e\right)(l+2e) + 5l. \tag{4}$$

### 5.1 Picking Machine Design

We begin by analyzing each of the three subsystems independently.

*Carousel Storage Subsystem:* A space requirement of 3,500 totes is needed in the picking machine's storage area. To determine the total throughput of the carousel storage system, we apply our expected cycle-time model from Appendix A. A storage subsystem with six carousels, each with 49 pick faces, results in an expected cycle time of 15.67 seconds per carousel. The total throughput of the storage subsystem is only 1,378 totes per hour (which does not satisfy our throughput requirement of 1,500 totes per hour). A storage subsystem with seven carousel systems, each with 42 pick faces, results in an expected cycle time of 15.60 and produces a total throughput of 1,615 totes per hour. Therefore a storage subsystem with seven carousels satisfies both the throughput and space requirements. This configuration requires 91 $m^2$ of floor space.

*Conveyor Subsystem:* The picking machine vendor sells systems with three conveyor velocities: 0.20 meters per second, 0.40 meters per second, and 0.50 meters per second. We apply the conveyor stability-condition model in Appendix B, resulting in system stability factors of 1.04, 0.52, and 0.41, respectively, for the three conveyor velocities. The 0.20 meters per second conveyor subsystem provides a stability factor greater than 1.0; therefore, this conveyor velocity will not be able to meet the throughput requirement. On the other hand, a conveyor velocity of 0.40 meters per second is selected because it results in a subsystem stability factor of less than 1.0 and will meet throughput.

*Pick Stations Subsystem:* The amount of time to process one tote consists of the pick time and a portion of the setup time. To meet the throughput requirement for our example, the number of pick stations is determined as 3 (i.e., $\lceil(1500 \times (3+30/15))/3600\rceil = 3$).

*Subsystem Utilization:* The subsystems of a picking machine do not work independently; thus, a picking machine's throughput is limited by the lowest throughput subsystem. In our example, the storage area produces the lowest throughput at 1,615 when the carousels are at 100% utilization. In this case, the pick station subsystem is operating at 75% utilization. Therefore, in our example providing additional pick station throughput capacity or conveyor velocity, without increasing the carousel system's throughput, will not impact the overall picking machine's throughput potential.

### 5.2 Carousel-Pod Design

Prior to calculating the throughput capabilities of a carousel-pod system, we need to use our inventory model from Section 3.1 to determine the space requirements. Initially, we assume that all pods have the same assortment of SKUs (i.e., $\alpha = 0$). A two-pod system requires 5,125 totes, a three-pod system requires 7,500 totes, and a four-pod system requires 10,000 totes, as calculated by (1). Using the model developed in [7], we determine that four pickers with four carousels per pod are required. This configuration results in a single carousel throughput of 94 totes per hour, a system throughput of 1,502 totes per hour, and a floorspace requirement of 500 m$^2$.

Next, we analyze a carousel-pod system where 500 of the lowest-demanded SKUs are not stored in multiple pods, but instead are stored in a single location. For a four-pod system, this represents the case when $\alpha$ equals 0.20. The inventory requirements of a two, three, and four pod system, as calculated using (2), are 4,625, 6,500, and 8,500 totes, respectively. All SKUs will not be stored in each carousel pod; thus, all orders will not be able to be fulfilled in their entirety from a single carousel pod. To accommodate the transportation of these orders among pods, we increase the average walk time from 2.0 seconds to 2.5 seconds. To meet the throughput and space requirement, four pickers with four carousels per pod are required, resulting in a system throughput of 1,614 totes per hour and a floorspace requirement of 428 m$^2$.

### 5.3 Design Comparison

Table 3 provides the requirements for the two technologies in terms of labor, inventory, storage infrastructure, and storage floorspace. For our example, the picking machine system has the lowest labor, inventory, storage infrastructure, and storage floorspace. Depending on labor and the overall infrastructure costs of the two systems, a cost analysis can be conducted to determine if the additional cost of a conveyor subsystem is warranted.

This example illustrates an additional advantage of a picking machine over a carousel-pod system. By decoupling the storage subsystem from the picking process, the heights of the carousels in a picking machine are not restricted by human reach capabilities. However, carousel-pod height is limited by human picker capabilities. This height advantage coupled with the increased inventory requirements of carousel-pod systems creates picking machine storage carousels that tend to be shorter in length than carousels in carousel-pod systems, which have floorspace advantages.

Table 3: Design Requirements Between a Picking Machine and a Carousel-Pod System

| | Picking Machine | Carousel-Pod System ($\alpha = 0$) | Carousel-Pod System ($\alpha = 0.20$) |
|---|---|---|---|
| Labor | 3 Operators | 4 Operators | 4 Operators |
| Inventory | 3,500 Totes | 7,500 Totes | 6,500 Totes |
| Storage Infrastructure | 7 Carousels | 16 Carousels | 16 Carousels |
| Storage Floorspace | 91 $m^2$ | 500 $m^2$ | 428 $m^2$ |
| Conveyor | Required | – | – |

## 6. Conclusions and Future Research

In summary, we analyzed inventory and throughput considerations of a complex order-fulfillment technology, the picking machine. We developed a probabilistic model capable of quantifying the inventory differences in a picking machine and a carousel-pod system. Through our analysis, we illustrated the inventory potential of using picking machine technology for the order-fulfillment of slow-moving or low-volume SKUs. We determined the throughput of a picking machine by analyzing each of its subsystems independently by developing a methodology that incorporated multiple analytical models. To analyze the storage system subsystem, we applied an expected single-command cycle time model for a carousel with an S/R machine performing batch retrievals. We determined if a picking machine's closed-loop conveyor is stable by applying an existing stability-condition model and determined the number of pick stations required to meet throughput. A case study example comparing a picking machine to a carousel-pod system was presented that illustrates how a manager could use our methodology to answer design questions.

Our research could be extended in the following ways. We assumed a carousel system was used as the storage area of a picking machine; therefore, a generalization of our work could be to incorporate different storage subsystems into our inventory and throughput models. We assume that pick stations for both picking machines and carousel-pod systems are equivalent, where the design of the pick station is actually a decision. Future research could analyze the impact of a pick station that does not enforce first-in-first-out processing of totes. Finally, we assume that a single SKU is stored in each tote, whereas some systems store multiple items per tote to increase storage density. Therefore, the trade off between increased storage density and throughput implications in such configuration could be explored.

## Appendix

## A. An Expected-Cycle Time Model for a Carousel System with an S/R Machine Performing Batch Retrievals

The following notation and definitions are excerpts of [13].

**Notation:**

| | |
|---|---|
| $H$ | the carousel's height |
| $L$ | the carousel's circumference |
| $v_h$ | the velocity of the horizontal carousel rotation |
| $v_v$ | the velocity of the S/R machine vertical travel |
| $t_v$ | the time to move the S/R machine from the I/O point to the top level of the carousel ($t_v = H/v_v$) |
| $t_h$ | the time to rotate the carousel one revolution ($t_h = L/v_h$) |
| $\tau$ | the normalization factor, which is the maximum of $t_h$ and $t_v$ |
| $b$ | the shape factor, defined as $b = min\{t_v, t_h\}/tau$ |
| $G$ | the handling time to pickup or discharge a tote |
| $g$ | the normalized handling time to pickup or discharge a tote, defined as $g = G/\tau$ |
| $m$ | number of pickfaces in the carousel |
| $n$ | batch size in number of totes |
| $X$ | a random variable that represents the horizontal rotation time |
| $A$ | the number of stops required to pick $n$ items from $m$ pickfaces |
| $E[CT]_n$ | the expected cycle time of a carousel with an S/R machine with a batch size of $n$ |
| $E[CT\|X=0]$ | the expected cycle time given no horizontal rotation is required |
| $E[CT\|X>0]$ | the expected cycle time given horizontal rotation is required |

Pazour and Meller [13] derive the expected-cycle time model for a carousel system with an S/R machine performing batch retrievals, which is provided in (5). The expected cycle times conditioned on whether horizontal rotation is required or not are shown in (6) and (7), respectively. The associated probabilities are shown in (8) and (9).

$$E[CT]_n = E[CT|X=0]Pr\{X=0\} + E[CT|X>0]Pr\{X>0\}. \tag{5}$$

$$E[CT|X>0] = \begin{cases} (E[R]^h + g)\tau & \text{for } \tau = t_h, \\ \\ (E[R]^v + g)\tau & \text{for } \tau = t_v. \end{cases} \tag{6}$$

$$E[CT|X=0] = (b+2g)\tau. \tag{7}$$

$$Pr\{X>0\} = \frac{A}{n}. \tag{8}$$

$$Pr\{X=0\} = \frac{n-A}{n}. \tag{9}$$

where $A = \left(\frac{m-1}{m}\right)\left[\sum_{k=0}^{m-1}(m-k)q_k(m,n)\right]$ and
$$q_k(m,n) = \binom{m}{k}\sum_{j=0}^{m-k}(-1)^j\binom{m-k}{j}\left(1-\frac{j+k}{m}\right)^n.$$

### B. A Conveyor Stability-Condition Model

The following notation and definitions are excerpts of [4]. In our analyses, we denote a queue that is formed by totes waiting to be placed on the conveyor as an *on-queue* and a queue that forms by totes being ejected off of the conveyor as an *off-queue*.

**Sets:**

| | |
|---|---|
| $C$ | Carousel Systems; indexed on $c; c = 1,2,\ldots,\lvert C\rvert$. |
| $O$ | Order-fulfillment or Pick Stations; indexed on $o; o = 1,2,\ldots,\lvert O\rvert$. |
| $Q$ | Conveyor on-queues; indexed on $q; q = 1,2,\ldots,\lvert Q\rvert$. |
| $K$ | Conveyor off-queues; indexed on $k; k = 1,2,\ldots,\lvert K\rvert$. |

**Parameters:**

| | |
|---|---|
| $\Delta_q$ | the flow rate on the conveyor that passes on-queue $q$ in loads per time unit. |
| $f_{qk}$ | the flow rate from on-queue $q$ to off-queue $k$. |
| $\chi^i_{qk}$ | 1 if a tote from on-queue $q$ to off-queue $k$ travels by queue $i$; 0 otherwise. |
| $V$ | the speed of the conveyor in the number of windows moved per time unit. |
| $\lambda_q$ | the arrival rate to on-queue $q$. |
| $\mu_o$ | the expected service rate at pick station $o$. |
| $\phi_o$ | the arrival rate to pick station $o$ off-queue. |

**Variables:**

| | |
|---|---|
| $SF_q$ | the stability factor for the conveyor segment that ends with on-queue $q$. |
| $SF_{sys}$ | the subsystem stability factor for the closed-loop conveyor. |

$$\Delta_i = \sum_{q\in Q, q\neq i}\sum_{k\in K} f_{qk}\chi^i_{qk} \quad \forall i \in Q. \tag{10}$$

$$SF_c = \frac{\lambda_c + \Delta_c}{V} \quad \forall c \in C. \tag{11}$$

$$SF^{on}_o = \frac{min\{\mu_o, \phi_o\} + \Delta_o}{V} \quad \forall o \in O. \tag{12}$$

$$SF_{sys} = \max_{q\in Q}\{SF_q\}. \tag{13}$$

The closed-loop conveyor is stable if and only if $SF_{sys} < 1$. Our notation maps to [4] directly for (10), (11), and (13). For (12), $\lambda_o = \min\{\mu_o, \phi_o\}$.

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