A MULTI-OBJECTIVE OPTIMIZATION APPROACH FOR DESIGNING AUTOMATED WAREHOUSES

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Abstract

A multi objective optimization of automated warehouses is discussed and evaluated in present paper. Since most of researchers in material handling community had performed optimization of decision variables with single objective function only (usually named with *minimum travel time*, *maximum throughput capacity*, *minimum cost*, etc.), the multi objective optimization (*travel time – cost – quality*) will be presented. For the optimization of decision variables in objective functions, the method with genetic algorithms is used. To find the Pareto optimal solutions, the NSGA II genetic algorithm was used. The main objective of our contribution is to determine the performance of the system according to the multi objective optimization technique. The results of the proposed model could be useful tool for the warehouse designer in the early stage of warehouse design.

1   Introduction

Warehouses with their basic purpose are an absolute necessity for a continuous and optimum operation of the production and distribution processes [1]. Warehouses are needed for various reasons, especially [1]: (*i*) to facilitate the coordination between the production and customer demand by buffering products for a certain period of time, (*ii*) to accumulate and consolidate products from various producers for combined shipments, (*iii*) to provide same-day delivery in production and to important customers, (*iv*) to support products customization activities, such as packaging, final assembly etc.

There are two categories of warehousing systems, named as mechanized warehousing systems (conventional warehouses) and automated warehousing systems (automated storage and retrieval systems).

Conventional warehousing systems are characterized by manually operated equipment managed and directed by a warehousing management system. Manually operated forklift trucks equipped with onboard terminals linked to a warehouse management systems, provide transportation, storage and retrieval (order picking) of transport unit loads. The onboard terminals display instructions to operators as well as providing them with the ability to communicate finished tasks. The presented technology increase efficiency by eliminating paper instruction and optimize the work routine of operators. The primary physical characteristics of the conventional warehouse facility are a low profile and therefore large floor area. Clear high is determined by the reach of a selected forklift truck. Traditionally the conventional warehouse facilities are less expensive and easier to build compared to automated warehouse facilities.

Today the conventional warehousing systems are characterized by automated guided vehicle (AGV) technology with automated forklift trucks managed and directed by a warehousing management system. Numerous manufacturers (Jungheinrich, Still, Linde, etc) offer automated forklift trucks (AFT). Automated forklift trucks utilize guide path technology in which an energized, floor-embedded wire created an electromagnetic field which activated sensors onboard the automated forklift to follow the path. A self guidance system provides the automated forklift with a free-roaming capability. ATF incorporate the primary features of manually operated forklifts including side shifting, sensors to determine the presence or absence of transport unit load at assigned interfaces, and the ability to adjust

vertical and horizontal positions of the forks to store or pick up the transport unit load.

Automated storage and retrieval systems (AS/RS) are characterized by high bay warehouses with automated storage and retrieval machine (S/R machine) which store and retrieve transport unit loads. System configurations include single- and multiple-deep and single- and multi-shuttle variants. All systems utilize S/R machine equipped with hoisted carriage which supports storage/retrieval shuttle tables or self powered rack entry modules. Shuttle equipped hoisted carriage support single- and multi-shuttle system; whereas self-powered rack entry modules support single- and multiple-deep system. Pick up/deposit queue conveyors usually at the front of rack structure generally provide an interface between storage and retrieval machine and a delivery system. This delivery system can be as simple as manually operated forklift trucks or as sophisticated as AGV. The disadvantage of those systems is non-adaptable to future changes and relatively high investment due to conventional warehousing systems.

Since the AS/RS had gained more attention of the material handling research community as the conventional warehousing systems, some previous work regarding design of automated systems will be briefly presented.

The design of automated warehouses has been studied by several authors. One of the first publications in the subject of optimizing the warehouses is represented by the work of Basan et al. [2], who have analyzed optimum dimensions of the warehouse, considering the chosen warehouse volume of the warehouse in dependence on various storage strategies. Karasawa et al. [3] have presented a design model of the AS/RS. In their work, the objective function is defined as non-linear and multi-variable, consisting of three main variables: (*i*) the number of storage and retrieval (S/R) machines, (*ii*) the length of the SR and (*iii*) the height of the SR; and also of constant values: cost of buying the land, cost of building the warehouse, cost of buying the storage rack (SR) construction and cost of buying S/R machines. The main disadvantage of this model [3] is that it refers only to the single command cycle. Ashayeri et al. [4] have presented a design model of the AS/RS that enables the determination of the main influential parameters when designing warehouses. Unlike Karasawa et al. [3], they have considered the warehousing operation of the dual command cycle. Bafna et al. [5] and Perry et al. [6] have used a combination of the analytical model and the system of discrete event simulations when designing the warehouse. Perry et al. [6] have used a special search method to determine optimum solutions for the AS/RS, which they have included in the simulation model

of the AS/RS. As a measure of the efficiency of the system, they have used the throughput capacity of the warehouse, in dependence on the number of S/R machines and the number of workplaces. The design of warehouses regarding to the influence of the storage policy has been presented by Rosenblatt and Roll [7]. When describing total costs, the authors have taken into account: (*i*) cost of building the warehouse, (*ii*) cost of buying storage equipment, (*iii*) costs arising from overloading the warehousing system (temporary shortage of the storage space) and (*iv*) costs that depend on a particular storage policy. An in-depth overview of the area of designing and controlling warehouses has been presented by Rouwenhorst et al. [8] in the form of the methodology of designing warehousing systems. The design process is presented with a structured approach, which takes into account the strategic, tactical and operational level of decision making. Gu at al. [9] have presented a comprehensive review of research on warehouse operation. Roodbergen and Vis [10] have presented a comprehensive explanation of the current state of the art in AS/RS.

Warehouse design according to optimization of the travel time, cost and quality considerations had been already considered, although researchers did not provide multi objective approach in their study. The described models refer only to the single objective optimization approach of AS/AR [3], [4], [5], [6]. The difference between approaches and models lies in the cost of elements included in the objective function, the decision on considered variables and the use of optimization techniques. Less has been done for multi objective optimization approach of automated warehouses, although studies on multi objectives have received a close attention in some references (Hwang at al. [11]; Steuer, R. E. [12]; Dev, K. [13]).

Beside mentioned papers, results of our research designing automated warehouses can also be found in [15], [16], [17], [18].

Generally, each warehouse involves a multidimensional problem. A design of the warehouse should therefore be managed with the consideration of (*i*) ***travel time***, (*ii*) ***cost*** and (*iii*) ***quality***. Successful warehouse designer should insure completion of the warehouse due to minimal travel time, minimal cost, and to the demanded quality specifications.

The purpose of our present paper is to analyse the design and optimization of the automated warehouse, according to multi objective optimization approach (*travel time − cost − quality*). The adopted approach is to apply multi objective function and discrete optimization in order to create the most efficient design of automated warehouses. Due to the non-linear, discrete, multi-variable and the most important multi objective function, the

heuristics method with genetic algorithms [19] is used. To find the Pareto optimal solutions, the NSGA II genetic algorithm is used [22]. The significant part of our research lies in the creation of a computer aided design tool for designing and optimizing automated warehouses according to multi objective optimization approach.

2   Model for designing automated warehouses

The primary purpose of this section is to present the optimization model which aims to resolve *travel time – cost – quality* tradeoffs problems. The model minimizes travel time, minimize cost and maximize quality of a warehouse according to project restraints and conditions.

Minimizing travel time

Travel time in most material handling facilities (in our case warehouses) relates to the movement of material handling devices like forklifts, S/R machines, etc. For the calculation of the mean single or dual command travel time, different approaches have been used. Some researchers are using analytical travel time models, while others are using discrete simulation. Travel time could be minimized by using efficient drives for faster movement and hoisting of the material handling devices in the horizontal/vertical direction. Beside the efficient drives, the length and the height of the warehouse (storage rack) should be in the appropriate relationship. The travel time is inversely depended from the throughput capacities. According to the values of the travel time and throughput capacities, the number of the material handling devices MHD (reach truck, very narrow aisle VNA truck, S/R machine) is defined. The objective is to minimize the travel time which is described as follows:

$$\text{function: } \min f_T\left(x_i\right); \quad i \in \left[1,10\right] \tag{1}$$

Minimizing cost

Cost is comparatively relative to travel time. Application of material handling devices with efficient drives (faster movement and hoisting) will no doubt increase the cost of the warehouse and the maintenance cost of material handling devices. For the relationship between cost and travel time, one can used a discrete function or continuous (linear/quadratic) function. The objective is to minimize the cost which is described as follows:

$$\text{function: } \min f_C\left(x_i\right); \quad i \in \left[1,10\right] \tag{2}$$

Maximizing quality

The quality could be defined in many ways. In proposed model, the quality is expressed with the number of material handling devices in the warehouse. If we have more than one material handling device in the warehouse, the probability that "*everything will be going well*" is higher than with only one material handling device. Thus, the quality actually represents the reliability of the warehouse. Quality is comparatively relative to cost. The objective is to maximize the quality which is described as follows:

$$\text{function: } \max f_Q(x_i); \quad i \in [1,10] \tag{3}$$

The independent variables with lower and upper bound, which are used in the above mentioned functions, are expressed as follows:

$$\text{real}: 0 < x_1, x_2, x_3, x_4, x_5, x_6, x_9, x_{10} \leq 1 \tag{4}$$

$$\text{integer}: 1 < x_7, x_8 < N \tag{5}$$

where x_1 refers to the selection of the MHD (reach truck, VNA truck, S/R machine), x_2 refers to the velocity of the MHD for driving in the horizontal direction, x_3 refers to the velocity of the MHD for lifting in the vertical direction, x_4 refers to the maximum lift height of the MHD, x_5 refers to the width of the picking aisle, x_6 refers to the quality (reliability) of the MHD, x_7 refers to the number of picking aisles, x_8 refers to the number of MHD, x_9 refers to the height of the warehouse, x_{10} refers to the length of picking aisles.

The proposed model consists of decision variables, operational parameters and costs of material handling devices, land and warehouse building. When designing the model, the following assumptions and notations had been applied:

- The warehouse is divided into picking aisles with SR on both sides; therefore there are double SR between the picking aisle and single SR along the warehouse walls. The I/O location of the warehouse is located on the lower, extreme left side of the warehouse (Figure 1).
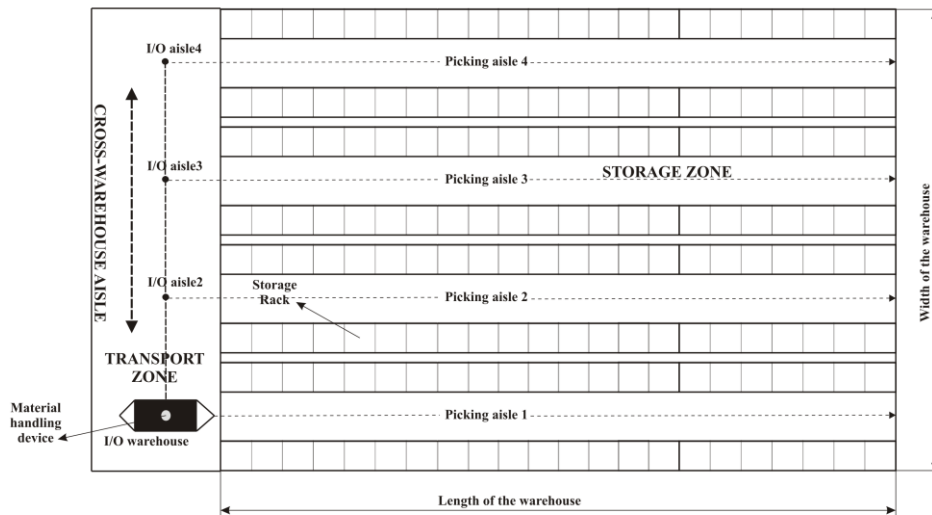
Figure 1: The layout of the warehouse

- The SR has a rectangular shape, whereby the I/O location of the SR is located on the lower left side of the SR (Figure 2).
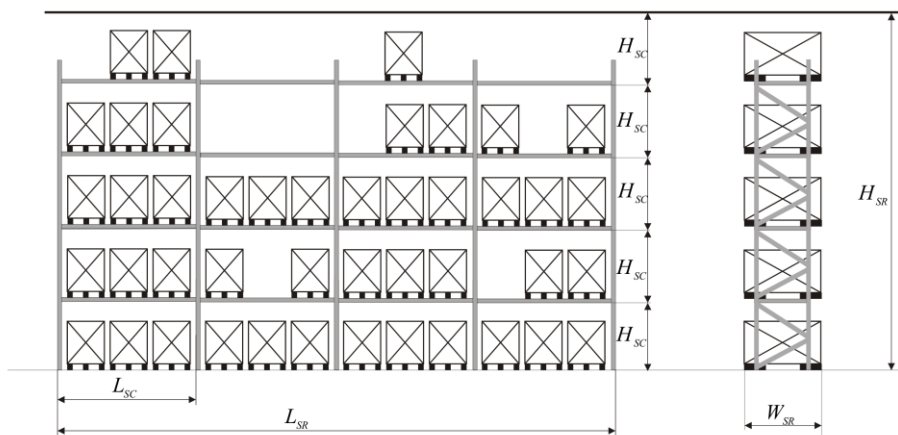

Figure 2: The side view of the storage rack

- There are three different types of MHD with its working width *Ast* (Figure 3).
- The number of MHD is less than or equal to the number of picking aisles ($n_{MHD} \leq R$).
- The MHD can travel in the cross-warehouse aisle, which enables access to adjacent picking aisles.
- The MHD enables the operation of SC, to which a variable share of travel time for travelling in the cross aisle must be added.

- Drive characteristics of the MHD velocity v, as well as the length L and height H of the SR are known.
- The length L and height H of the SR are large enough for the MHD to reach its maximum velocity v_{max} in the horizontal direction and in vertical direction.
- Randomized storage is used, which means that any rack opening in the storage compartment is equally likely to be selected for the storage or retrieval assignment.
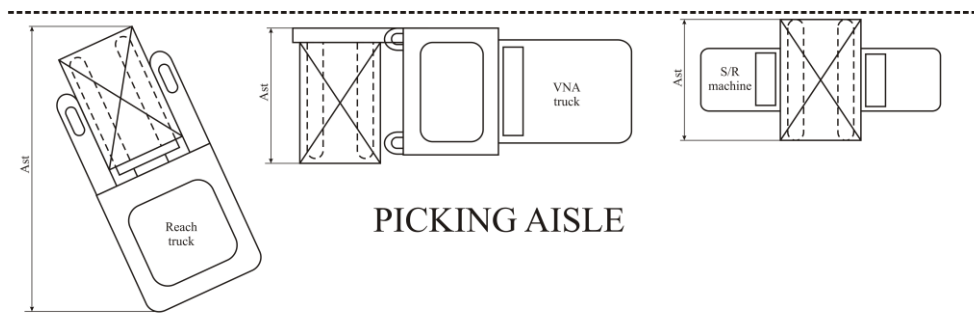

Figure 3: Material handling devices used in the proposed model

Also the following notation is introduced:

AS/RS	automated storage and retrieval systems
Ast	working width of the picking aisle
S/R	storage and retrieval
SR	storage rack
SC	single command
T(SC)	mean single command travel time
T(TBA)	travel between aisle time component
MHD	material handling device
VNA	very narrow aisle
TUL	transport unit load
I/O$_{aisle}$	input/output location of the picking aisle
I/O$_{war}$	input/output location of the warehouse
b	shape factor
q	warehouse volume
$f(x_i)$	objective function
x_i	variable
L_{WAR}	length of the warehouse
L_{SR}	length of the storage rack
L_{SC}	length of the storage compartment

H_{WAR}	height of the warehouse
H_{SR}	height of the storage rack
H_{SC}	height of the storage compartment
W_{WAR}	width of the warehouse
W_{SR}	width of the storage rack
W_{SC}	width of the storage compartment
T	time of one shift
n_{MHD}	number of MHD
R	number of picking aisles
λ	throughput capacity
n	number of TUL in storage compartment
v	velocity
v_{max}	maximum velocity
v_x	velocity in the horizontal direction
v_y	velocity in the vertical direction
t_x	travel time in the horizontal direction
t_y	travel time in the vertical direction
I_{MHD}	investment for MHD
I_{LAND}	investment for land
I_{WAR}	investment for building
C_{SRM}	maximum cost of MHD per piece
C_{MHD}	normalized cost of MHD per piece
C_{LAND}	cost of land per square meter
C_{WAR}	cost of building per cube meter
Q_{MHD}	quality of material handling device

The proposed model is represented with a mathematical model, which includes decision variables, all relevant operational, physical parameters, and investment costs and will be detail discussed in the following section.

3   Definition of the design model

3.1 Travel time definition

Proposed model is based on the single command cycle. The operation of the SC encompasses either the storage or the retrieval sequence. The SC in the warehouse combines travelling of the selected MHD in the cross warehouse aisle and in the picking aisle. The efficiency of the SC is based on: (*i*) travelling of the selected MHD in the picking aisle *i* and (*ii*) travelling of the selected MHD to adjacent picking aisle in the cross-warehouse aisle.

- ***Travelling of the MHD in the picking aisle i***

Under travelling of the selected MHD in the ith picking aisle, the selected MHD is capable of visiting a single storage or retrieval location. Let the storage (or retrieval) point be represented by $P(x, y)$ where $0 \leq x \leq L$ and $0 \leq y \leq H$.

For the S/R machine and VNA truck, the travel time t_{xy} from the I/O$_{aisle(i)}$ location will be $t_{xy} = Max(t_x, t_y)$, where t_x is the horizontal travel time and t_y is the vertical travel time. For the reach truck, the travel time t_{xy} from the I/O$_{aisle(i)}$ location will be $t_{xy} = t_x + t_y$, where t_x is the horizontal travel time and t_y is the vertical travel time. For calculation of the mean single command travel time for the S/R machine and VNA truck, the FEM 9.851 had been used [21]. For calculation of the mean single command travel time for the reach truck, the modified FEM 9.851 had been used [21].

The calculation of the mean single command travel time for the S/R machine and VNA truck is calculated as follows:

Time for travelling in the horizontal direction to point P_1

$$t_{xP_1} = \frac{L}{5v_x} \tag{6}$$

Time for moving in the vertical direction to point P_1

$$t_{yP_1} = \frac{2H}{3v_y} \tag{7}$$

Time for travelling in the horizontal direction to point P_2

$$t_{xP_2} = \frac{2L}{3v_x} \tag{8}$$

Time for moving in the vertical direction to point P_2

$$t_{yP_2} = \frac{H}{5v_y} \tag{9}$$

The maximum travel time in selected picking aisle i from I/O point to point P_1 in the horizontal or in the vertical direction, is equal to the next expression:

$$t_{P_1} = 2MAX\left(t_{xP1}, t_{yP1}\right) \tag{10}$$

The maximum travel time in selected picking aisle i from I/O point to point P_2 in the horizontal or in the vertical direction, is equal to the next expression:

$$t_{P_2} = 2MAX\left(t_{xP2}, t_{yP2}\right) \tag{11}$$

The calculation of the mean single command travel time for the reach truck is calculated as follows:

Times t_{xP_1}, t_{yP_1}, t_{xP_2}, t_{yP_2} equals above expressions (6), (7), (8) and (9).

The maximum travel time in selected picking aisle i from I/O point to point P_1 in the horizontal and in the vertical direction, is equal to the next expression:

$$t_{P_1} = 2\left(t_{xP_1} + t_{yP_1}\right) \tag{12}$$

The maximum travel time in selected picking aisle i from I/O point to point P_2 in the horizontal and in the vertical direction, is equal to the next expression:

$$t_{P_2} = 2\left(t_{xP_2} + t_{yP_2}\right) \tag{13}$$

The mean single command travel time in selected picking aisle i

The mean single command travel time $T(SC)_{picking\ aisle}$ in selected picking aisle i, becomes:

$$T\left(SC\right)_{picking\ aisle} = \frac{1}{2}\left(t_{P_1} + t_{P_2}\right) \tag{14}$$

- ***Travelling of the MHD in the cross-warehouse aisle***

Travel between aisles time component (TBA) corresponds to travelling of the selected MHD from I/O$_{war}$ location to i^{th} picking aisle.

$$T\left(TBA\right) = \frac{W_{war}}{v_x} \tag{15}$$

- ***The mean single command travel time in the warehouse***

The mean single command travel time for the selected MHD is represented with the following expression:

$$T\left(SC\right) = T\left(SC\right)_{picking\ aisle} + T\left(TBA\right) \tag{16}$$

The throughput capacity λ of the warehouse for the selected MHD is represented with the following expression:

$$\lambda = \frac{T}{T\left(SC\right)} \cdot n_{MHD} \tag{17}$$

3.2 Quality definition

The quality is defined according to next expression:

$$Q = Q_{MHD} \cdot n_{MHD} \tag{18}$$

Q_{MHD} indicates the quality of MHD; n_{MHD} indicates the number of selected MHD.

3.3 Cost definition

The investment in buying material handling devices per piece I_{MHD}:

$$I_{MHD} = C_{MHD} \cdot n_{MHD} \cdot C_{SRM} \tag{19}$$

C_{MHD} indicates the normalized cost of the selected MHD; n_{MHD} indicates the number of selected MHD; C_{SRM} [EUR] indicates the maximum cost of MHD (in our case S/R machine that can serve more than one picking aisle).

The investment in buying the land per square meter I_{LAND}:

$$I_{LAND} = L_{WAR} \cdot W_{WAR} \cdot C_{LAND} \tag{20}$$

L_{WAR} indicates the length of the warehouse [m]; W_{WAR} indicates the width of the warehouse [m]; C_{LAND} [EUR/m^2] indicates the cost of the land per square meter.

The investment in building the warehouse per cube meter I_{WAR}:

$$I_{BUILDING} = L_{WAR} \cdot W_{WAR} \cdot H_{WAR} \cdot C_{WAR} \tag{21}$$

L_{WAR} indicates the length of the warehouse [m]; W_{WAR} indicates the width of the warehouse [m]; H_{WAR} indicates the height of the warehouse [m]; C_{WAR} [EUR/m^3] indicates the cost of the building per cube meter.

Total cost TC:

$$TC = I_{MHE} + I_{LAND} + I_{BUILDING} \tag{22}$$

4   Pareto optimization design

Although single objective optimization problems may have a unique optimal solution, multi objective problems offer a possibly uncountable set of solutions, which when evaluated produce vectors whose components

represent trade-offs in decision space. A decision maker then implicitly chooses an acceptable solution by selecting one of these vectors.

The objective used in our contribution is to optimize *travel time – cost – quality* which is formulated as a multi objective problem. When optimizing decision variables x_i, $i \in [1,10]$ one must take into account certain constraints referring to: (1) geometrical constraints of the warehouse, (2) the minimum required Q of the warehouse and (3) the throughput capacity has to be higher than or equal to the required throughput capacity, (4) the number of MHD has to be lower than or equal to the number of picking aisles ($n_{MHD} \leq R$).

To search for an optimal trade of among *travel time – cost – quality*, the NSGA II genetic algorithm was used [22]. The algorithm is designed for solving multi objective problems. The output of the algorithm is a large number of solutions lying on or near the Pareto optimal frontier.

To find the Pareto optimal solutions, the design procedure used in present model is explained as follows:

1) Initialization

- Parameters initialization on optimization model: set the number of objectives; set the number of constraints; set the number of independent and dependant variables.

- Parameters initialization on genetics algorithm: set the size of population; set the number of generations; set the probability of crossover operation; set the probability of mutation.

2) Forming Pareto front

Travel time, *cost* and *quality* for every solution are computed in P(*t*) according to equations (1), (2) and (3). Next, the population P(*t*) is sorted based on the non-domination algorithm into each front F(*t*) in criterion space. Individuals in first front are given a fitness value of 1 and individuals in the second front are given a fitness value 2 and so on. The first front is also called Pareto front which will include the best solutions.

3) Genetic operation

Genetic algorithm (GA) is a procedure for searching optimized objective functions by the principles of natural genetics and natural selections. The main operation is related to *selection*, *crossover* and *mutation* (Figure 4).
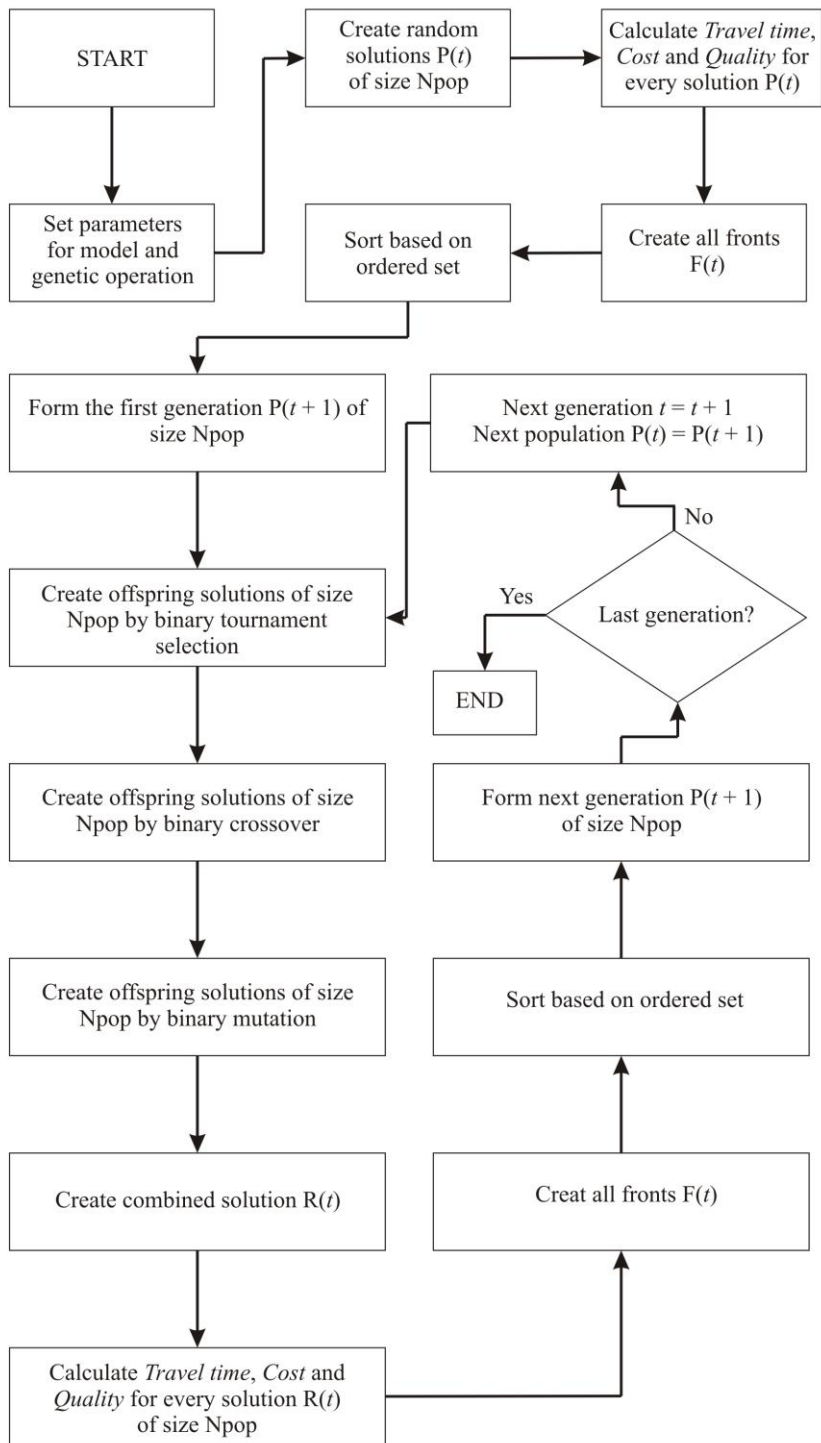
Figure 4: A Pareto optimization algorithm

5 Illustrative example and simulation results

In this section an illustrative example and simulation results are presented. With the optimization of decision variables x_1, x_2, x_3, x_4, x_5, x_6, x_7 in the proposed model, the optimal design of warehouse is defined. The input data for this example are based mainly on information from practice and sales representatives of companies supplying the warehouse equipment. With regard to the following project constraints: the length of the warehouse/storage rack L_{SR} (1 − 100) m and the height of the warehouse/storage rack H_{WAR} (1 − 18) m, the warehouse volume $q_{min} = 1000$ TUL, the length of the storage compartment $L_{SC} = 2800$ mm, the height of the storage compartment $H_{SC} = 1600$ mm, the depth of the storage compartment $W_{SC} = 1200$ mm, the number of TUL in the storage compartment $n = 3$, the number of aisles $R_{MAX} = 100$, the maximum cost of MHD $C_{MHD} = 250.000,00$ EUR, the cost of land per square meter $C_{LAND} = 5,00$ EUR/m^2, the cost of building per square meter $C_{WAR} = 10,00$ EUR/m^2. Operational parameters, parameters of material handling devices and costs all in normalized values, are presented in following tables.

Table 1: Independent variables with lower and upper bounds

Variables	Lower bound	Upper bound
variable x_1, **Reach truck**		
variable x_2, horizontal velocity v_x	0,1	0,4
variable x_3, vertical velocity v_y	0,1	0,3
variable x_4, maximum lift height h_3	0,1	0,4
variable x_5, aisle width Ast	0,9	1,0
variable x_6, quality Q	0,1	0,4
variable x_1, **VNA truck**		
variable x_2, horizontal velocity v_x	0,4	0,7
variable x_3, vertical velocity v_y	0,4	0,7
variable x_4, maximum lift height h_3	0,4	0,8
variable x_5, aisle width Ast	0,5	0,9
variable x_6, quality Q	0,4	0,7
variable x_1, **S/R machine**		
variable x_2, horizontal velocity v_x	0,7	1,0
variable x_3, vertical velocity v_y	0,7	1,0
variable x_4, maximum lift height h_3	0,8	1,0
variable x_5, aisle width Ast	0,1	0,5
variable x_6, quality Q	0,7	0,9
Other independent variables		
variable x_7, number of picking aisles R	1	N

variable x_8, number of MHD n_{MHD}	1	N
variable x_9, height of the warehouse H_{WAR}	1	N
variable x_{10}, length of the storage rack L_{SR}	0,1	1,0

The lower and upper bound of variables in *Table 1* are expressed with normalized values that can be easily changed and have no impact on model structure or optimization process. The values presented in *Table 1* demonstrate one selected study case.

Table 2: Dependent variables with lower and upper bounds

Input data	Lower bound	Upper bound
Reach truck		
cost of buying *reach truck* C_{RT}	0,1	0,3
VNA truck		
cost of buying *VNA truck* C_{VNAT}	0,3	0,6
S/R machine		
cost of buying *S/R machine* C_{SRM}	0,6	1,0
Other dependent variables		
Cost of warehouse C_{WAR}	0,1	1,0

The lower and upper bound of input data in *Table 2* are expressed with normalized values that can be easily changed and have no impact on model structure or optimization process. The values presented in *Table 2* demonstrate one selected study case.
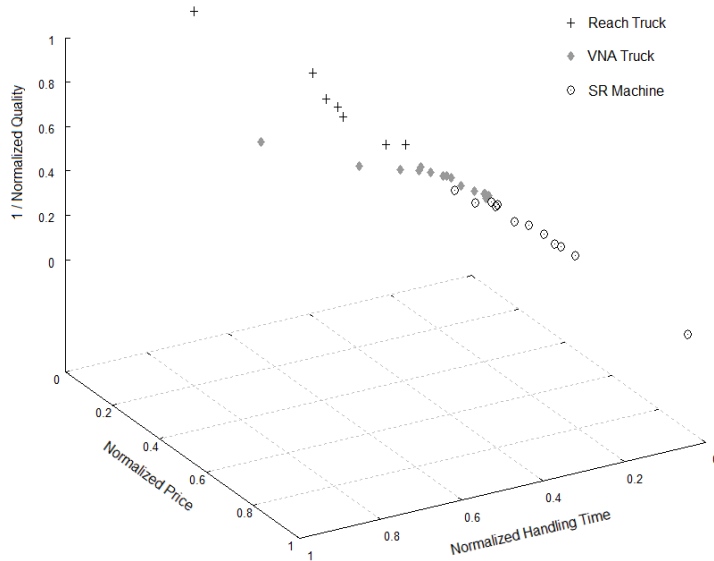

Figure 5.1: The first Pareto front after 5 generations (smaller is better)
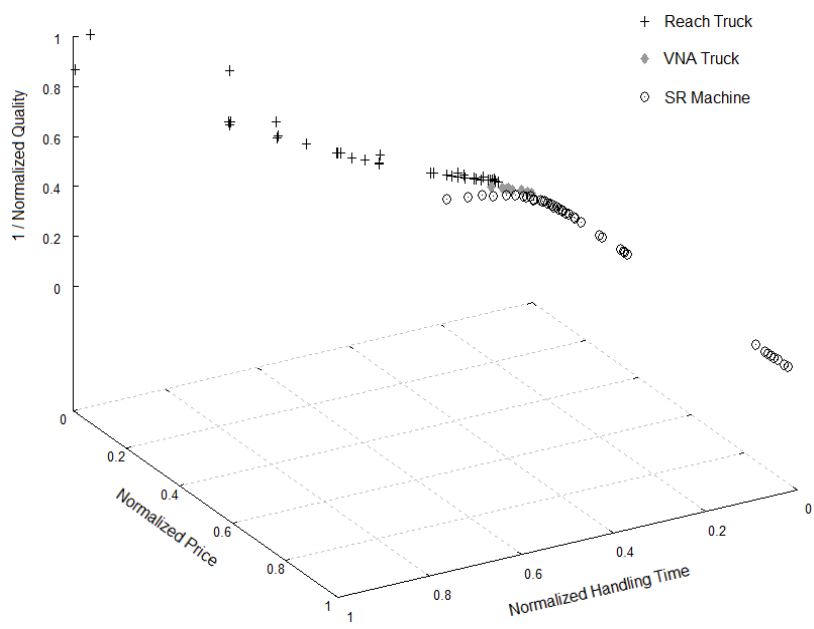
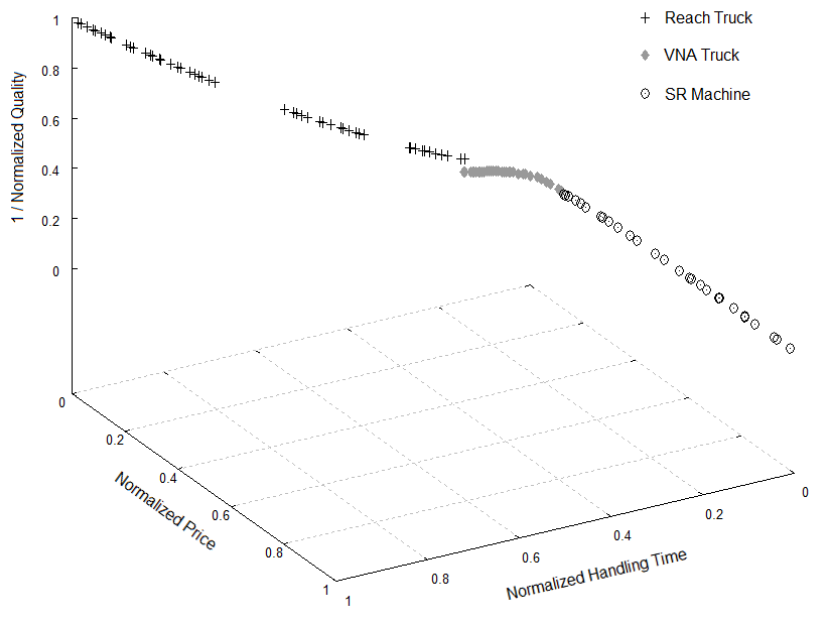Figure 5.2: The first Pareto front after 20 generations (smaller is better)

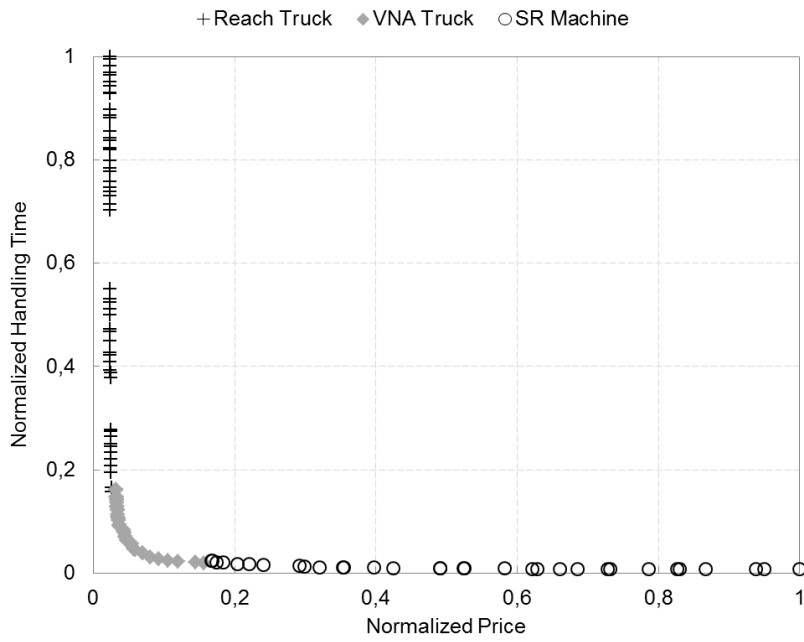Figure 5.3: The final Pareto front after 200 generations (smaller is better)

Figure 5.4: The final Pareto front in terms of normalized handling time and
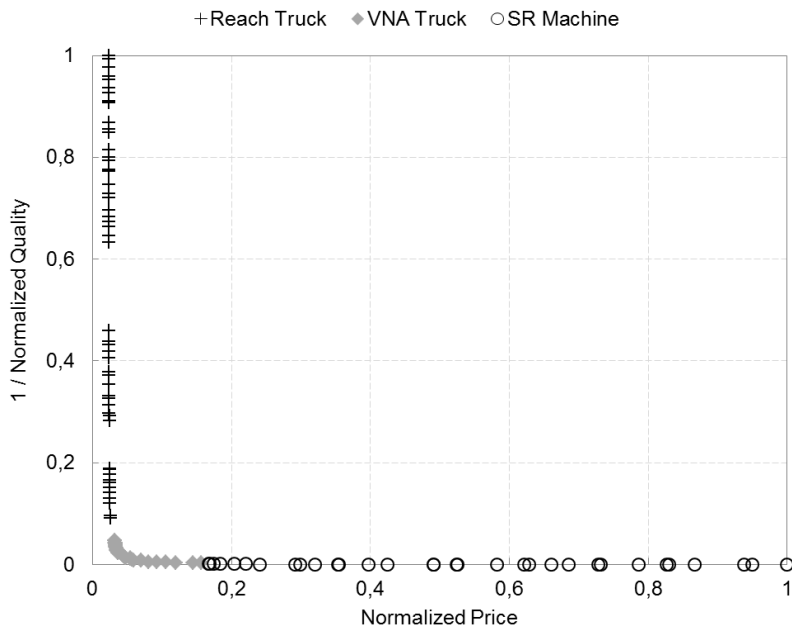normalized price

Figure 5.5: The final Pareto front in terms of inversed normalized quality
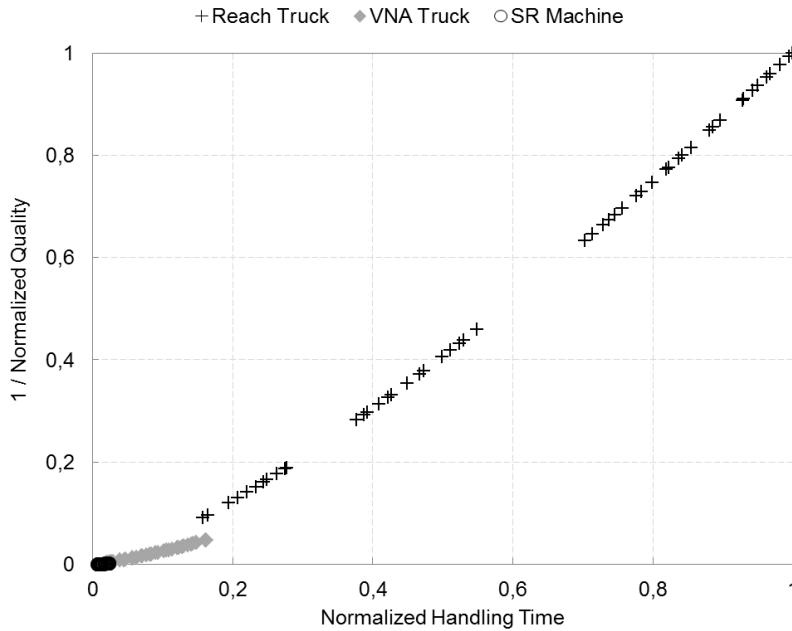and normalized price

Figure 5.6: The final Pareto front in terms of inversed normalized quality
and normalized handling time

Figures 5.1, 5.2, 5.3, 5.4, 5.5, 5.6 show the results of Pareto optimal front of decision variables x_i, $i \in [1,10]$ in the proposed model according to 5th, 20th and 200th generation. For the optimization of decision variable, the NSGA II algorithm had been used [22]. The primary reason for using the evolutionary algorithm is its ability to find Pareto optimal solution in a simulation run.

The optimization of decision variables x_i, $i \in [1,10]$ was carried out according to the following evolutionary and genetics operators: *the degree of crossover was set to* 0.9; *the degree of mutation was set to* 0.1; *the size of population was set to* 100; *the number of generations was set to* 200. Values of *crossover* and *mutation degrees* were chosen in accordance with our extensive analyses and experience of researchers who have been engaged in the development and application of the GA method. The size of population depends greatly on the number of decision variables, which indirectly influences the necessary number of generations. Due to the proposed decision variables x_i, $i \in [1,10]$ the comprehensive analyses has indicated that in most cases the GA finds Pareto optimal front already with 200 generations.

Diagram on Figure 5.1 shows that GA after 5th generation form a chosen number of random designs of the warehouse for the selected type of MHD (reach truck, VNA truck and S/R machine). Warehouses that do not follow the required constraints, defined at the optimization of decision variables, are deleted and not considered in next generations. This means that with the increase in the number of generation, the good solutions are continually eliminated and replaced by better solutions. In this way the Pareto optimal solutions can be found. The number of randomly chosen designs of the warehouse is the same as the size of the population n or in most cases smaller than n.

Diagram on Figure 5.3 illustrates after 200th generation that warehouses marked with different symbols for reach truck, VNA truck and S/R machine, present the Pareto optimal frontier of solutions. The response to the optimization of decision variables x_i, $i \in [1,10]$ in the proposed model, indicate the optimal investment (cost) for the warehouse due to selected material handling device (travel time) and reliability (quality) of the system.

The Pareto optimal frontier of solutions could be very useful information for the warehouse designer in the early stage of warehouse design. According to the results in diagrams on Figures 5.4, 5.5, 5.6 a single solution from the Pareto optimal frontier could be selected as the representative solution for the warehouse design. For example, if the expected throughput capacity is high and there is no limitation in height, one could choose the S/R machine as the representative material handling device. The selection of the S/R machine has impact on high quality and cost. In this case the investment in the warehouse could be excessive in comparison with the selection of VNA truck or reach truck.

Opposite, if the expected throughput capacity is low and there is a limitation in height, one could choose the VNA truck or reach truck as the representative material handling device. In the same way, according to the results in diagrams on Figures 5.4, 5.5, 5.6, a single solution from the Pareto optimal frontier of solutions could be selected as the representative solution for the warehouse design. The selection of the VNA truck or reach truck as the representative material handling device has impact on good quality and relatively low cost. In this case the investment in the warehouse is moderate in comparison with the selection of S/R machine.

6   Conclusions

In this paper, multi objective optimization approach of designing automated warehouses is presented. Due to the high complexity in designing and optimizing modern warehouses, the conventional design process rises at higher and more demanding levels, in the form of the computer aided design and optimization.

The proposed design model is based on the structured approach and refers to pallet storage system with several picking aisles. Unlike the single objective optimization problem, the multi objective optimization problem has not been used a lot in warehousing design process. The essential part in the proposed model is the application of three objective functions named *travel time − cost − quality*. The objective functions are represented with a mathematical model, which includes decision variables x_i, $i \in [1,10]$, all relevant operational and physical parameters, investment and operating costs. Due to the non-linearity, discrete shape of objective functions and proposed decision variables, the method of NSGA II [22] GA has been applied in order to optimize decision variables. The usefulness of the proposed model was presented in a case study involving the design of a warehouse. The results of the proposed model could be useful information for the warehouse designer in the early stage of warehouse design.

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