Robust Design of Public Storage Warehouses

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Abstract

We apply robust optimization and revenue management in public storage warehouses. We optimize the expected revenue of public storage warehouses against the worst cases with a max-min revenue objective, and the decision variables are mainly the number of storage units for each storage type. With the robust design, we can observe worst-case revenue improvement.

1 Introduction

Public storage warehousing is a mature industry in the U.S. and a fast-growing business in Europe and Asia Pacific. As the SSA (Self Storage Association, the official association of this industry in the U.S.) reported, the number of such facilities has climbed from only 6,601 facilities at year-end 1984 to over 46,500 at year-end 2010. In the year of 2010, the total revenue of this industry in the U.S. was $22 billion (USD). This business has become one of the fastest-growing sectors of the United States commercial real estate industry over the last 35 years (see [1]). The industry in Europe had boomed after a breathtaking growing from 2006 to 2009. According to the 2008 industry annual report of the SSA UK (Self Storage Association of the UK, the official association of this industry in the UK), the number of public storage warehouse facilities in Europe had sharply increased from 2007 to 2008, particularly in those emerging markets. For example, the industry had increased by 200% in Poland, 117% in Switzerland, 64% in Denmark (see [2]).

Public storage warehousing usually operates as self-storage mode and thereby its cost is low and stable. For public storage warehouses with an objective to maximize the expected revenue and with a stable cost level, revenue management rather than cost control is critical. A typical public storage warehouse contains storage spaces of different storage types, each type with a specific number of storage units. A customer rents one or multiple storage units of an appropriate type for several months. However, the existing storage types or the available number of storage units for each type may not fit the needs of the market. The number of available storage units of some types may be insufficient, while other types are abundant. This results in either lost customers and revenue, or inefficient utilization of capacity of one type, which also may bring potential loss in another type.

The main facility design question is to provide a facility design improving the expected revenue that is with a better fit between storage design (types and numbers) and market demand. Based on data of warehouses in America, Europe and Asia, Gong et al.[3] propose a facility design approach for public storage warehouses with three different cases: an overflow customer rejection model, and two models with customer upgrade possibilities: one with reservation and another without reservation. They analyze models for real warehouse cases mainly by stochastic models and dynamic programming (see [3], [4]).

While these methods assume a probability distribution or a specific stochastic process of demands, it is difficult to accurately estimate the probability distribution of the total arriving storage requests or associated revenue in some warehouses. To provide a method to solve the practical problem, we present robust optimization models to handle problems with uncertain marketing data and parameters which cannot be accurately estimated.

Robust optimization is a developing literature body recently, which plays an important role in revenue management (see [5]). Robust optimization is a method for modeling optimization problems under uncertainty, to find optimal decisions for the worst-case realization of the uncertainties within a given set. Typically, the original uncertain optimization problem is converted into an equivalent deterministic form (called the robust counterpart) using strong duality arguments and then solved using standard optimization algorithms. Soyster [6], Ben-Tal and Nemirovski [7], and Bertsimas and Sim [8] describe the method for constructing robust counterparts for uncertainty sets of various structures. Robust optimization has been applied in various fields including supply chain management, finance, and logistics. Goh and Sim [8] develop ROME, an algebraic modeling toolbox designed to solve a class of robust optimization problems in the MATLAB environment, to solve application problems. AIMMS provides robust optimization add-on, with the ability of handling with multi-periods optimization problems without falling into the trap of “the curse of dimensionality”.

Revenue management (see [10]) is a main concern of public storage warehouses. Robust optimization plays an important role in revenue management (see [4]). We optimize the expected revenue of public storage warehouses against the worst cases with a max-min revenue objective, and the decision variables are mainly the number of storage units for each storage type. This research is mainly in the field of facility design (see [11]). Rosenblat and Lee[12] propose a robust approach to facility design. This paper makes contribution to facility planning (see [11]) and proposes a facility design method to handle risks and uncertainties and with an objective to increase revenue.

2 Models

The main research problem is to determine how many storage type i units should be constructed to fit demand. There are m different storage type units with a specific integer storage size area $c_i, 1 \le i \le m$. We assume that the random arrival processes of customers requesting a storage type i unit, $1 \le i \le m$ are independent Poisson processes with arrival rates λ_i and that the occupation times of a storage type i unit are independent and identically distributed random variables with expected occupation time β_i. Customers requesting a storage type i unit are called type i customers.

When a type i customer arrives and storage type i units are not fully occupied, the customer will be accepted at a price of r_i per period for a storage type i unit. When all storage type i units are occupied, customers who initially are interested in a storage type i unit, may accept with probability p_i to pay a price of r_{i+1} for a storage type $i+1$ unit. A customer willing to accept this is called an upscaled customer. If a storage type $i+1$ unit is available, an upscaled customer will be served, otherwise the customer is lost.

Let $x_i, 1 \le i \le m$ be the number of storage type i units built for type i customers at level i and $y_i, 1 \le i \le m-1$ the type $i+1$ units reserved for type i customers upscaled to level $i+1$. A type i customer who finds upon arrival all the x_i storage type i units occupied may choose to be upscaled and use one of the y_i reserved units, if one is available. If all y_i units are occupied, the customer is lost. Customers of type m finding upon arrival all x_m units busy are directly lost. A customer of type $i+1$ is not allowed to occupy one of the y_i units reserved for upgraded type i customers. Each rejected type i customer is willing to upscale with probability p_i.

To analyze this model we first look at the process followed by each type of customers separately. The long-run average revenue obtained from type i customers can be split in the long-run average revenue obtained from the x_i and y_i units. The number of occupied storage type i units among the available x_i can again be modeled by a queueing loss system. Due to our assumption of exponentially distributed occupation times this is an $M/M/x_i/x_i$ loss queue. The long-run average revenue obtained from the x_i units is given by $\sum_{i=1}^{m} r_i \rho_i (1 - B(x_i, \rho_i))$. The long-run average revenue obtained from the y_i units is $\sum_{i=1}^{m-1} r_i \eta_{i+1}(x_i) \beta_i \left(1 - P_r\left(x_i, y_i\right)\right)$, where $\eta_{i+1}(x_i) = p_i \lambda_i B(x_i, \rho_i)$ and $P_r\left(x_i, y_i\right)$ the rejection probability that an upscaled type i customer will find all the reserved y_i units occupied. Let Λ be the set of demand data for all storage types of D quarters in the examining years, we calculate the worst performance as

$$\min_{\lambda_d \in \Delta, d=1,\ldots,D} \left[\sum_{i=1}^{m} r_i \rho_i (1 - B(x_i, \rho_i)) + \sum_{i=1}^{m-1} r_i \eta_{i+1}(x_i) \beta_i \left(1 - P_r\left(x_i, y_i\right)\right) \right]$$

Given the price r_i for each storage type i unit per unit of time, the motivation of robust design is to minimize the loss in revenue due to variation in demand, and to decide how many units of each type to build (and reserve) such that we can find the best one among the worst performances, and we present a robust model as follows,

$$\max \quad \left\{ \min_{\lambda_d \in \Delta, d=1,\ldots,D} \left[\sum_{i=1}^{m} r_i \rho_i (1 - B(x_i, \rho_i)) + \sum_{i=1}^{m-1} r_i \eta_{i+1}(x_i) \beta_i \left(1 - P_r\left(x_i, y_i\right)\right) \right] \right\} \ldots (R)$$

$$s.t. \quad \sum_{i=1}^{m-1} (c_i x_i + c_{i+1} y_i) + c_m x_m \le C$$

$$x_i, y_i \in \mathbf{Z}_+$$

3 Analysis

We apply the following random search robust optimization algorithm to calculate the design and solve model (R).

Algorithm: robust optimization for PS warehouses

(I) For $t = 1:T$.
(I.1) Generate a sample x^t *from* $U\left(\underline{x}, \overline{x}\right)$ *with consideration of the capacity constraint* $\sum_{i=1}^{m} c_i x_i \le C$.
(I.2)(A) For $d = 1:D$, *compute the objective value* $R_d(x^t)$ *from the* d^{th} *demand pattern.*
(I.2)(B)Compute the worst performance of this generated design x^t *in* D *demand patterns* $R^t = \min\left(R_d(x^t), d = 1:D\right)$.
(II)Compute $R^* = \max\left(R^t, t = 1:T\right)$
(III) Return $\boldsymbol{x}$ *corresponding to* R^* *as the robust design.*

We take the S.P. Rotterdam warehouse as an example, to illustrate how to provide a robust design, such that a warehouse can achieve the best revenue performance among the worst revenues from demand data of D quarters.

Table 1: Parameters of S.P. Rotterdam warehouse

Items	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8
Types(m^2)	3	6	9	12	15	18	22	27
Price (Euro)	109	132	177	225	254	372	436	468
Old design	34	44	58	25	18	27	3	4

We use optimal results from Table 3 in [3] , which is by a sensitivity analysis of S.P. Rotterdam warehouse design based on data of 8 quarters, to construct the search range $\left[\left(\underline{\mathbf{x},\mathbf{y}}\right), \left(\overline{\mathbf{x},\mathbf{y}}\right)\right]$ for problem (R). For iterations $t = 1:T$, we generate t samples $(\mathbf{x},\mathbf{y})^\tau, \tau = 1,\ldots,t$ from the search range while considering the capacity constraint. For each of the samples $(\mathbf{x},\mathbf{y})^\tau$ we compute its revenues for D different demand patterns, and obtain the worst value $R^\tau = \min\left\{R_d\left((\mathbf{x},\mathbf{y})^\tau, \lambda_d\right), d = 1,\ldots,D\right\}$ for this design sample. The optimal revenue corresponding to the robust design is now $R(t)^* = \max\left\{R^\tau, \tau = 1,\ldots,t\right\}$ for iteration t. We can then return the $(\mathbf{x},\mathbf{y})$ value corresponding to $R(t)^*$ as the robust design for iteration t. The theoretical optimal robust revenue is $R^* = \lim_{t\to\infty} R(t)^*$.

We present the searching process $\{R(t)^*, t = 1:T\}$ in Fig.-1. By setting the convergence criterion as $|R(t+1) - R(t)| / R(t) \leq 0.3\%$, we observe the search converges in the last 500 iterations. We therefore calculate the average value from the last 500 values $\{R(t)^*, t = 2500:3000\}$, and obtain the robust optimal revenue value as 43912, which is a significant improvement compared with the current worst revenue value 32622 based on the current design. We use the design obtained after 3000 iterations {50(6), 9(15), 19(16), 46(0), 2(15), 25(5), 9(4), 19(0)} as the robust design.

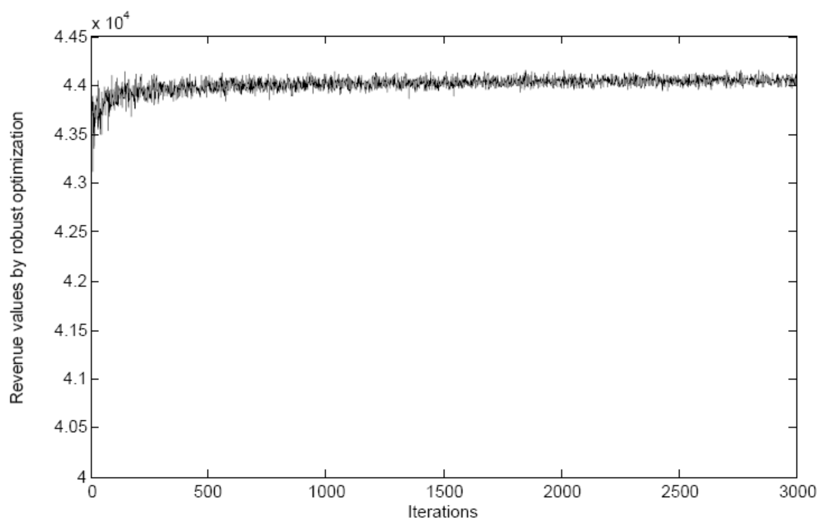

Figure 1: Robust optimization for S.P. Rotterdam warehouse

4 Concluding remarks

This paper presents a robust design method for public storage warehouses by applying robust optimization and revenue management. In regard to available information, it needs to further consider two situations: (1) Some warehouses maintain incomplete market data. They cannot accurately estimate customer demand, including average order arrival rates. But they can roughly provide a scope of customer demand by history data. This happens in small and middle facility providers and some large-scale companies with relatively unqualified information management. (2) Some warehouses can provide more market data. They can estimate customer demand information, including average order arrival rates. They cannot accurately specify some parameters of their market demand models; instead they can roughly provide a scope of parameters by historical data. This happens in some world-class facility providers (like Shurgard) and some middle-scale companies with good information management systems.

Methods and technologies of robust optimization need improvement in the further research. The limitation of random search robust optimization algorithm is in searching speed and accuracy. It is interesting to explore better algorithms for robust design of large–scale public storage warehouses.

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