ROBUST MATERIAL HANDLING SYSTEM DESIGN BASED ON THE RISK VERSUS COST TRADEOFF

Marc Goetschalckx
Georgia Institute of Technology

Edward Huang
George Mason University

Pratik Mital
Georgia Institute of Technology

Abstract

The design and planning of major material handling systems belongs to the class of systems design problems under uncertainty. The overall structure of the system is decided during the current design stage, while the values of the future conditions and the future planning decisions are not known with certainty. Typically the future uncertainty is modeled through a number of scenarios and each scenario has an individual time-discounted total system cost. The overall performance of the material handling system is characterized by the distribution of these scenario costs. The central tendency of the cost distribution is almost always computed as the expected value of the distribution. Several alternatives can be used for the dispersion of the distribution such as the standard deviation and variance. In this study the standard deviation of the cost distribution is used as the measure of the risk of the system. The goal is to identify all configurations of the material handling system that are Pareto-optimal with respect to the tradeoff between the expected value and the standard deviation of the costs; such Pareto-optimal configurations are also called efficient. The final selection of the material handling system for implementation can then be made based on the Pareto graph and other considerations such as the risk preferences of the system owner.

1   Introduction

1.1   System Design under Uncertainty

Material handling systems are an essential component of any facility that produces, consumes, or transforms one or more goods. Prominent and traditional examples are manufacturing plants and distribution systems. Material handling systems are also important components of service facilities such as hospitals, restaurants, and retail malls. Material handling systems range from the very ad hoc manual handling systems, such as restocking the shelves in a grocery store, to the complex and hardware intensive systems, such as fully automated distribution centers for the wholesale of frozen meats. Especially the design and management of large and automated material handling systems exhibits all the characteristics of designing engineered systems under uncertainty. During the design phase, the structure of the system and the management policies of the system are determined. However, at the design time the future conditions are not known with certainty. The uncertainty of the future is typically captured in a number of possible scenarios. For a particular scenario, the structure of the system and the values of all the uncertain parameters are assumed to be known, so the performance of the system under that scenario can be computed. Material handling systems are typically required to be feasible, i.e. to satisfy one or more service requirements, for the parameter values of the scenario and have cost minimization as their performance objective.

The cost can be divided into two categories: one set of cost is associated with the structure of the material handling system and the second set is associated with the behavior or execution of the material handling system. All of the cost components associated with the material handling system are included in the total system cost. Many material handling systems have a system life time of many years. In such cases, the time when the costs are incurred has to be incorporated explicitly in the evaluation of the system performance. For large material handling system, the performance objective is then the minimization of the net present value of the total system cost. The net present value computation discounts future costs to the design time.

The expected value of the net present values of the scenarios gives the expected efficiency or reward of the system. However, the uncertainty of the future also has to be considered. The standard deviation of the net present values of the scenarios gives the risk of the system. In the next section we will discuss the tradeoff between the system risk and reward.

1.2   Multi-objective Performance Evaluation: Risk versus Reward Tradeoff

The performance as measured by the total system cost of a material handling system is a stochastic variable at its design time. In other words, the total system costs associated with each scenario for a particular material handling system yield a cost distribution. One goal of this research is the computation of the first and second moment of this distribution, i.e. the expected value and standard deviation. The first moment of the cost

distribution is nearly universally used as the measure for the central tendency of the distribution. Systems with a smaller expected value of the costs are preferred. The smaller expected value of the costs is denoted as the "reward" of the system when using the cost minimization objective. The dispersion of the costs is often interpreted as a measure of risk associated with the system; see for example the ISO standard on risk management [1]. Common measures of dispersion used are the variance and standard deviation. The classic example is the use of the variance in the construction of equity portfolios by Markowitz [2]. More complex measures of dispersion such as the Value at Risk (VaR) and the Conditional Value at Risk (CVar) are used in the financial industry but are rarely used for the evaluation of material handling systems; see Fishburn [3].

Since there are at least two measures of performance of interest to the system designer, a tradeoff between them is generally required, i.e. the risk versus reward tradeoff. A material handling system configuration can now be evaluated by its two performance characteristics, the expected value and standard deviation of the total system cost for the different scenarios. Assuming the performance measure is being minimized, a material handling system configuration is said to be efficient if no other configuration has the same or smaller expected value and the same or smaller standard deviation; see e.g. Kung at al. [4] for the definition of efficient discrete alternatives. An efficient configuration is also said to be Pareto-efficient or Pareto-optimal. Specifically, the objective of the design process is to find all Pareto-optimal configurations with respect to the expected value and standard deviation of the scenario performance measure. The configurations can be plotted in a risk analysis graph with the expected value on the horizontal axis and the standard deviation on the vertical axis. The set of all the system configurations dominated by a Pareto-efficient system configuration corresponds to a rectangle in the risk analysis graph. Figure 1 shows a risk analysis graph with three Pareto-optimal configurations and their dominance regions. The risk tradeoff was based on 5 scenarios (N=5) and the configuration with the lowest expected cost dominates nearly all other configurations. In this example, only configurations with the lowest or near-lowest standard deviation are not dominated by the configuration with the lowest expected cost. Also note that in this example, the optimal configuration for the mean value of all the parameters, i.e. the mean-value problem (MVP), is strongly dominated by nearly all other configurations.

The second goal of this research is develop an efficient algorithm that identifies all the Pareto-optimal configurations for a material handling system. This allows the configurations to be plotted in the risk analysis graph followed by the selection of the final preferred alternative based on the risk versus reward preferences of the corporation that will own the material handling system.

2   Material Handling System Design Models

2.1   Review of Material Handling Research

Motivated by the prevalence of material handling systems and by their significant costs, a large number of system design models and systems design approaches for material handling systems have been developed. A comprehensive review of these research results is beyond the scope of this paper. Both digital simulation and optimization have been used to find material handling systems with desirable performance characteristics. Early design methods based on digital simulation were proposed by Perry [6] and Ashayeri et al. [5]. Some of the earliest research in the area of optimization of warehousing systems was reported in Gudeshus [7] and Kunder and Gudehus [8]. Comprehensive reviews of the analysis and design of material handling systems haven created by Rouwenhorst et al. [9] and Gu et al.[10].

One of the most comprehensive models for the design and evaluation of automated storage and retrieval systems (AS/RS) is developed in Lerher and Srami [11]. Because the model is non-linear integer in function of the main dimensions of the AS/RS, they use a genetic algorithm to find the optimal.

Despite the large body of research and the variety of models and solution algorithms used, several observations can be made with respect to the design methodology. In the vast majority of the research on material handling systems design, the configuration of the systems is based on either a simulation or symbolic model. For the subset of results based on symbolic models, nearly all design models are deterministic, i.e. they use a single value for each of the parameters. The simulation models are inherently stochastic and heuristic. Typically they focus on a homogeneous set of system configurations. Especially at the conceptual design stage, when the type and configuration of the material handling system is determined, models that explicitly incorporate the uncertain future and the stochastic nature of the parameters do not appear to exist. This is even more the case for models that more than a single stochastic performance measure. To our knowledge, a design model that that makes the tradeoff between risk and reward does not exist.

A second observation is that often the recurring operational costs are simply added to the one-time construction costs. The acceptance or ranking criterion often used is the payback period. However, for major material handling systems with productive life times of many years, methodology based on the time-discounted value of money such as the net present value is required. In the following, we develop a design methodology that explicitly incorporates the uncertainty, is based on the net present value, and finds all the Pareto-optimal configurations for the risk versus reward tradeoff.

3   Robust Systems Design Problem

In the following a framework is presented to design an engineered system while explicitly incorporating the uncertainty of the future through a set of scenarios and by finding all the Pareto-optimal configurations that tradeoff the risk and reward of the designed system. The final selection of the system to be implemented can then be made based on the risk preferences of the corporation and other considerations. Examples of

such considerations are the total investment cost of the system and the time required to implement and bring the system to an operational state.

The framework is decomposed in a general master problem that makes the risk versus reward tradeoffs and a domain-specific sub problem that determines the optimal configuration of the system and the corresponding cost using scenario-dependent data parameters. This structure makes the application of the design framework general and adaptable to different material handling systems or even systems in general. Applying the framework to a different type of system only requires creating the domain-specific sub model.

3.1   Warehouse Design Sub Model

The design and operation of a material handling system and its performance is dependent on the type of the material handling system and on a large number of technology parameters. We focus our attention here on the design of a storage and retrieval system for unit loads. A common implementation of this type of system is a single-deep, pallet load automated storage and retrieval system or ASRS. A schematic of a unit-load storage and retrieval system (UL-SRS) is given in Figure 2. We present the model for this material handling system in a top down manner. At level 0 of the model, a verbal formulation describes the overall objective, constraints, and decisions. Other high level constraints can be added if relevant to a particular design project.

Formulation 1: Level 0 Design Model for Unit Load Storage and Retrieval System

Objective*: minimize sum of time-discounted total annual system costs*

Subject to*: warehouse storage capacity*
warehouse throughput capacity
warehouse maximum footprint size
warehouse maximal height
warehouse maximal width
warehouse maximal depth
maximal investment budget

Decisions*: warehouse external layout (external dimensions and locations of docks)*
warehouse internal layout (# aisles, rack height, rack depth, I/O point locations)
warehouse operational policies (storage zones, storage policies, handling policies)

Many models have been developed for this type material handling system. One of the most recent and comprehensive is given in Lerher and Srami [11]. The model fundamentally decides the number of aisles, the height of each storage rack, and the depth of each storage rack. It includes many detailed calculations created by required clearances and required structural elements. At level 1 of the design model, these clearances and structural elements are ignored. To simplify the notation, the level 1

model shown next corresponds to a single scenario and the scenario subscript s has been omitted. The following notation is used in the level 1 model (again without scenario subscript).

Parameters

H planning horizon expressed in years, subscripted by t

df discount factor for one year (may be modeled as dependent on the year t)

p index of a particular planning time interval, e.g. peak, weekly, monthly, every Monday

$f_t()$ function to compute total system cost for a particular year t

I_{tp} inventory of unit loads during year t and planning interval p

$SPOL_{tp}$ storage policy, i.e. where to place unit loads, during year t and planning interval p

$HPOL_{tp}$ handling policy, how to put away and retrieve unit loads, during year t and planning interval p

RD rack depth, number of unit load storage locations perpendicular to the travel aisle in one rack face (most often 1, but 2 or more are possible)

SLD depth of a storage location for a single unit load perpendicular to the travel aisle including all clearances

SLW width of a storage location for a single unit load along the travel aisle including all clearances

SLH height of a storage location for a single unit load along the travel aisle including all clearances

TAW width of a single travel aisle

$MAXW$ maximum warehouse width (dependent on parameters such as the land lot)

$MAXD$ maximum warehouse depth (dependent on parameters such as the land lot)

$MAXH$ maximum warehouse height (dependent on parameters such as zoning and construction norms)

NO_{tp} required number of handling operations in year t during planning interval p (throughput requirement)

α_{tp} fraction of the number of handling operations that is executed with dual command cycles (i.e. a put away followed by a retrieval operation on a single trip)

NSC_{tp} number of single command cycles executed in year t during planning interval p

NDC_{tp} number of single command cycles executed in year t during planning interval p

T_{tp} time duration of planning interval p in year t in compatible units with t_{SCtp} and d_{SCtp}, e.g. the length of one shift expressed in minutes (480 minutes)

$f_{tsc}(\ )$ function to compute the average single command cycle time

$f_{tdc}(\ )$ function to compute the average dual command cycle time

ρ_{tp} availability of the material handling system during planning interval p in year t

Decision Variables

z_t total system cost for year t

NA number of two-sided aisles

NC number of columns of storage locations in a rack face

NL number of levels (rows) of storage locations in a rack face

WW warehouse width

WD warehouse depth

WH warehouse height

t_{SCtp} average single command cycle time in year t during planning interval p

t_{DCtp} average dual command cycle time in year t during planning interval p

Formulation 2: Level 1 Design Model for Unit Load Storage and Retrieval System

$$\min \quad \sum_{t=1}^{H} \frac{1}{(1+df)^t} z_t \tag{1}$$

$$s.t. \quad z_t = f_t\left(t, NA, NC, NL, RD, I_{tp}, SPOL_{tp}, HPOL_{tp}\right) \tag{2}$$

$$I_{tp} \leq Q \tag{3}$$

$$Q = 2 \cdot NA \cdot NC \cdot NL \cdot RD$$

$$NA \cdot \left(2 \cdot RD \cdot SLD + TAW\right) \leq WW$$

$$NC \cdot SLW \leq WD \tag{4}$$

$$NL \cdot SLH \leq WH$$

$$WW \leq MAXW$$

$$WD \leq MAXD \tag{5}$$

$$WH \leq MAXH$$

$$NSC_{tp} = NO_{tp} \cdot (1 - 2\alpha_{tp})$$

$$NDC_{tp} = \alpha_{tp} \cdot NO_{tp}$$

$$NSC_{tp} \cdot t_{SCtp} + NDC_{tp} \cdot t_{DCtp} \leq NA \cdot T_{tp} \cdot \rho_{tp} \tag{6}$$

$$t_{SCtp} = f_{tsc}\left(t, NA, NC, NL, RD, I_{tp}, SPOL_{tp}, HPOL_{tp}\right)$$

$$t_{DCtp} = f_{tdc}\left(t, NA, NC, NL, RD, I_{tp}, SPOL_{tp}, HPOL_{tp}\right)$$

In the above formulation, three functions have not been fully specified. The function to compute the total cost is a summation of the annual fixed costs associated with the material handling equipment and the construction of the rack and the annual variable cost associated with the equipment usage. This usage depends on the average cycle times for the single and dual command cycles. These average cycle times in turn depend on the dimensions of the rack face, the location of the rack I/O points, the storage policy, the handling policy, and the intensity of the use of the material handling system. To the best of our knowledge no closed form formulas exist to compute the average cycle times except for the simplest case such as random storage. For an extensive discussion of cycle times in unit load storage systems see Gu et al. [12] and Vasili et al. [13]. A level 1 approximation of the cycle times is computed according to the norms established by FEM [14] as follows. The total inventory during the planning interval is assumed to be divided evenly among the aisles. The average single command cycle time is computed as the round-trip travel time from the aisle I/O point to the point P1 with coordinates $\left(X/6, 2Y/3\right)$ of the rack face or $\left(NC \cdot SLW/6, \, 2 \cdot NL \cdot SLH/3\right)$ plus two I/O operations.

The average dual command cycle time is computed as the round-trip travel time from the aisle I/O point to point P1, then to point P2, and then returning to the I/O point plus four I/O operations. The coordinates of point P2 are $\left(2X/3, Y/6\right)$ of the rack face or $\left(2 \cdot NC \cdot SLW/3, \, NL \cdot SLH/6\right)$. These equations are linear in the decision variables. Obviously these formulas represent a significant simplification of the travel time model. However, travel times are a factor in the computation of the marginal costs, which in turn are incorporated into the annual costs, which are discounted over time. All of these costs are averaged over the different scenarios.

It should be noted that the warehouse design sub problem using the simplified travel time models is a mixed-integer non-linear programming formulation with cubic constraints. This problem can be transformed into a mixed-integer programming problem with quadratic constraints by complete enumeration either over the number of aisles or over the number of levels in a storage rack. The resulting quadratic constraints have a convex region and can be handled by the latest versions of some of the commercial MIP solvers. Alternatively, the problem can be transformed into a mixed-integer programming problem with linear constraints by complete enumeration over all combinations of the number of aisles and the number of levels in the rack. Both of those numbers are bound from above by practical considerations. Computationally, the upper bound was set to 25. Either solution method can be executed depending on the capabilities and efficiency of the mixed-integer programming solver available.

3.1   Risk Tradeoff Master Model

The warehouse design sub model assumes that the values of the parameters are known. In the master model, the tradeoffs caused by this uncertainty are explicitly considered. The warehouse design sub model is always solved for a particular scenario. For that scenario the values of the parameters are assumed to be known. For notational simplicity

the scenario index was omitted from the warehouse model. Only the master problem deals with uncertainty and multiple scenarios.

It should be noted that this decomposition approach is not a two stage decision model. There is only a single decision maker and decisions are only made at the design time. This allows the decision maker to control the variability of the performance of the warehouse across multiple scenarios. Conceptually the decision maker computes a plan for every possible eventuality (scenario) and stores the plan. In the future, the warehouse observes which scenario has come to pass, retrieves and then executes the plan for that scenario. As a result, no planning decisions are made after the initial design phase.

Finally, the risk tradeoff master model is not specific to the design of material handling systems and in general is formulated as a maximization problem. Hence, the objective of the warehouse sub problem has to be transformed to the maximization of the negative of the costs. This negative of the cost will be denoted by the "profit" for that particular scenario and warehouse configuration.

The mathematical formulation of the single stage warehouse design problem is denoted as the Single-Stage Mean-Standard Deviation Robust Design Problem or SS-MSD-RDP. This formulation will be solved by an iterative optimization algorithm described in the next section that systematically examines the full range of tradeoff ratios between the expected value and the standard deviation of the scenario profits. This ratio is denoted by κ. For a given value of κ the SS-MSD-RDP (κ) problem shown below is solved. The solution consists of the optimal configuration of the warehouse Y, the optimal warehouse plan for each individual scenario, s, and the associated profit z_{sY} for that plan. The profit z_{sY} is the objective function value of the embedded secondary optimization problem, i.e. the warehouse design sub problem, which is not shown in the formulation below. z_{sY} is bounded from above by z_{sY}^{MAX}, which is the maximum profit that can be achieved under scenario s for a specific configuration of the warehouse denoted by Y. The upper bound of z_{sY}^{MAX} on the scenario profit encapsulates all the constraints of the tactical planning problem. In other words, it possible to operate the warehouse so that it yields a profit of z_{sY} for configuration Y and scenario s as long as this profit is no larger than z_{sY}^{MAX}.

Formulation 3: Single-Stage Mean-Standard Deviation Robust Design Problem (SS_MSD_RDP) Formulation

$$\max \quad Exp(Y) - \kappa \cdot StdDev(Y)$$
$$s.t. \quad Exp(Y) = \sum_s p_s \cdot z_{sY}$$
$$StdDev(Y) = \sqrt{p_s \left(Exp(Y) - z_{sY} \right)^2}$$

$$z_{sY} \leq z_{sY}^{MAX}$$

$$Y \text{ binary}, \ z_{sY} \text{ continuous}$$

An efficient optimizing algorithm has been developed to solve the SS-MSD-RDP based on the adaptation of the branch-and-reduce algorithm by Ryoo and Sahinidis [15]. It systematically investigates the full range of the tradeoff ratio κ from the maximum expected profit to the minimum expected variability. The solution algorithm solves the corresponding SS-MV-RDP problem with the variance of the profits in the objective function and transforms the solution to the standard deviation domain.

4   Numerical Experiment

Numerical experiments are currently being conducted to estimate the run time of the design framework. The tests are performed on a computer with two Xeon processors, 24 GB of RAM and running under Windows 7 (64-bit). The solver used for the unit load storage model is CPLEX 12.2. For the initial experiments, the warehouse sub model requires no more than 20 seconds. The overall design model requires less than 20 minutes. At this time no parameter tuning has been performed and the solvers are used with their default parameters. A complete numerical experiment is planned.

Many questions with respect to the tuning of the algorithm are topics for future research. The warehouse design sub model is currently being solved as mixed integer formulation with quadratic constraints inside a complete enumeration loop. It can be further reduced to a mixed-integer linear formulation inside two-level enumeration loop. The impact of solving fewer quadratic constraint MIP formulations versus solving more linear constraint MIP formulations has to be further investigated.

4   Conclusions

The framework developed above finds all the Pareto-optimal configurations for complex engineered systems explicitly considering the tradeoff between the expected value of the profit distributions (the reward) and the dispersion of the profit distributions as measured by the standard deviation (the risk). The Pareto-optimal configurations can be plotted in a risk analysis graph. Final selection of the system configuration can then be made by the corporation based on its risk preferences. The total computation times for the creation of the risk analysis graph are less than hour. This is an acceptable running time for a design algorithm and is significantly shorter than the data preparation time for scenarios.

A conceptual graph is shown in Figure 3. In addition to the expected values of the costs, the 3σ boundary curves are also drawn. In this conceptual case the 3σ interval for the minimum cost warehouse configuration does not overlap the 3σ interval for the warehouse configuration with minimum variability of the costs. In other words, with near 100 percent certainty implementing the minimum cost warehouse configuration will

yield lower costs than implementing the minimum variability configuration. So the later would not be a reasonable selection. In real-world design projects, making such inferences may not always be possible since the 3σ intervals of various configurations may overlap, but some probabilistic confidence statements would still be possible, e.g. there is a less than 5% change that configuration A will cost more than configuration B.

The framework uses a significant simplification of the travel time model in the warehouse. In addition, many of the algorithm tuning decisions have to be investigated further. Both areas provide rich opportunities for further research. Finally, the risk analysis framework can be adapted to different and more complex material handling and warehousing systems design problems. In other to evaluate a different technology or operation mode only the corresponding warehouse design sub problem has to be constructed and formulated as an optimization problem. Equipment manufacturers may develop the models for the systems they provide and over time a database of such models can be created. The framework then would allow the comparison of the risk and reward of very different technologies and implementations.

Acknowledgements

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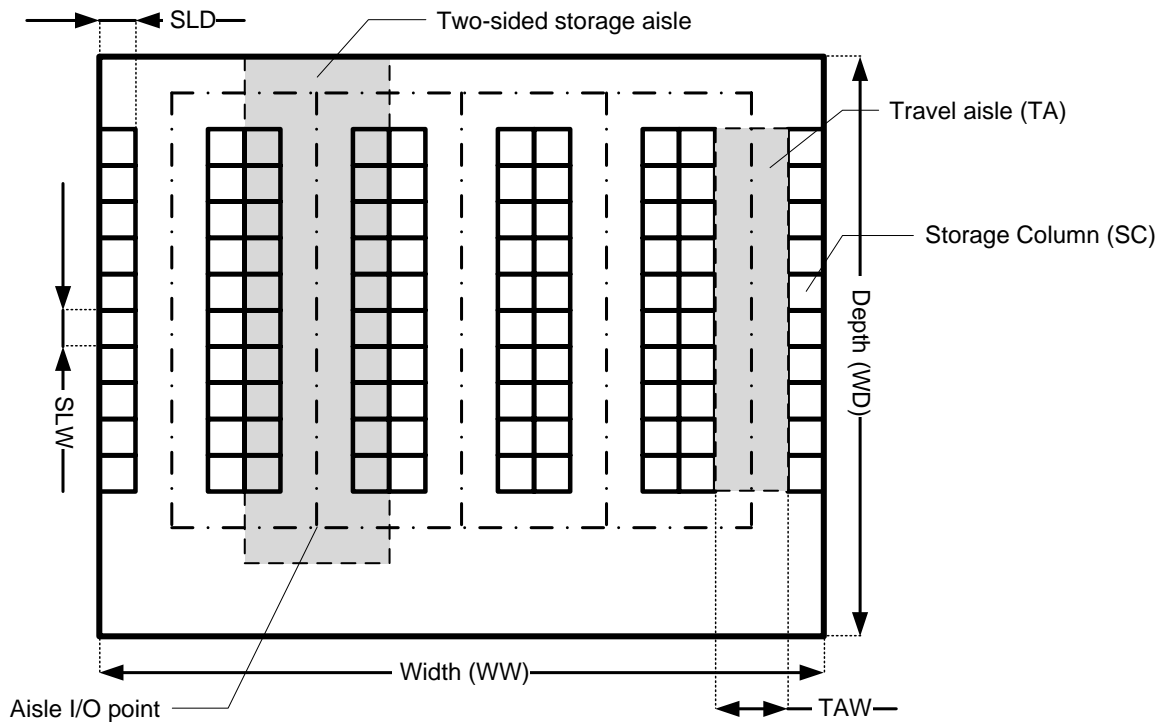Figure 1: Risk Graph with Pareto-Optimal Configurations.

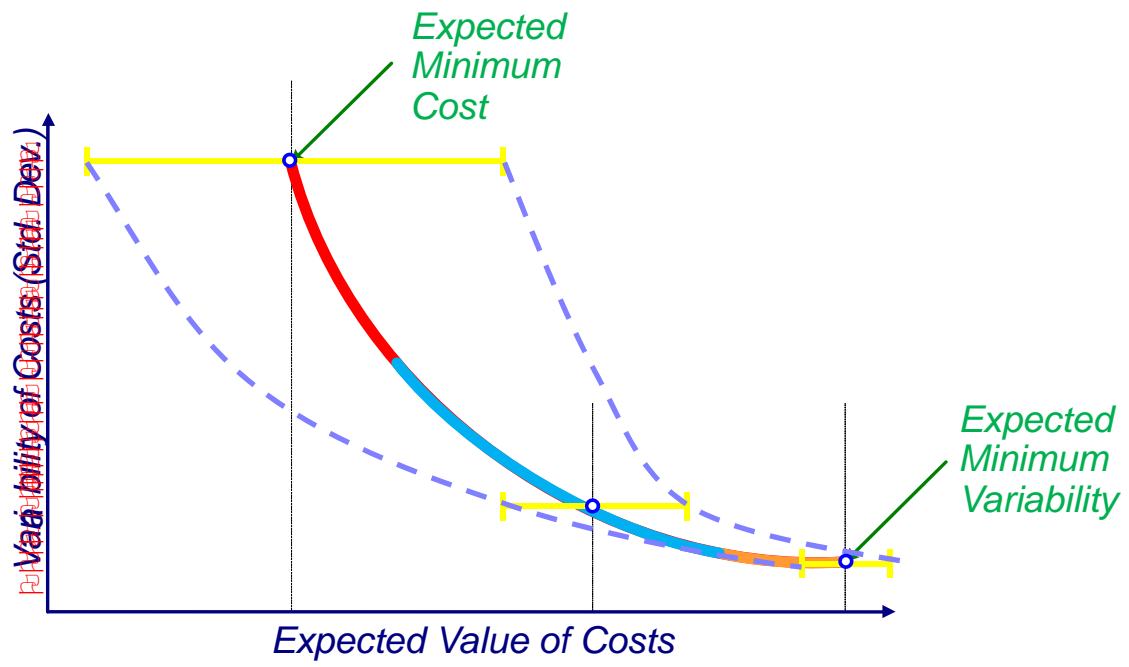Figure 2: Schematic of a Unit Load Storage and Retrieval System.

Figure 3: Risk Analysis Graph with Probability Boundary Curves.