DISTRIBUTION PLANNING
CONSIDERING WAREHOUSING DECISIONS

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Abstract

Modern supply chains heavily depend on warehouses for rapidly fulfilling customer demands through retail, web-based, and catalogue channels. The traditional approach that considers warehouses as cost-centers has affected the profitability of numerous supply chains. A lack of synchronization between procurement and allocation decisions causes warehouses to scramble for resources during peak times and be faced with under-utilized resources during drought times. Warehouses, however, have emerged as service-centers and it is imperative that warehousing decisions be an integral part of supply chain decisions. In this paper we propose a mixed-integer programming model to integrate warehousing decisions with those of inventory and transportation to minimize long-run distribution cost. Preliminary experiments suggest a sizeable reduction in the level and variance in the warehouse workforce requirements. A cost savings ranging between 2-6% is also realized.

1 Motivation

According to the 20th State of the Logistics Report [5], logistics costs comprise of 9.4% of the U.S. GDP, which accounts to about $1,309B dollars. Warehousing costs rose almost 10% from 2007 to 2008 to $122B dollars across 600,000 small and large warehouses in the nation. Warehouses, however, are often considered as cost centers and treated outside the realms of supply chain planning and optimization. Consequently, warehouse managers are often squeezed between their procurement department and the allocation department (or stores). The procurement department determines the quantity of products to be purchased from vendors and subsequently stocked at the warehouse to reap maximum benefits from quantity discounts. The centralized allocation department (or decentralized store ordering) determines the quantity to be delivered from warehouses to stores in order to minimize the inventory and/or transportation costs. Both these decisions often cause a large variation in the inbound and outbound shipments at the warehouse resulting in an imbalance in warehouse's workload. Warehouse managers often scramble for resources during peak-times resulting in hiring temporary workers and/or paying overtime, and have trouble generating enough work during slow times resulting in underutilized resources.

The problem we consider was motivated by our general observation in industry and specifically at the warehouse of our industry partner --- an U.S-based apparel supply chain. The warehouse at this apparel supply chain operates in a reactive mode; that is, warehousing decisions are made after the procurement and allocation decisions. Consequently, depending on the timing and quantity of products received by and shipped from the warehouse, the workforce utilization varies significantly. We observed that during a 5-day week the workforce utilization varied from 50% to 150%, a staggering 300% variation. This has cost the company millions of dollars annually due to operating inefficiencies at the warehouse. This begs the question, *how would a supply chain benefit if it proactively accounted for warehousing decisions at the planning stage, instead of warehouses having to react?*

To address this question, we introduce the *integrated warehousing-inventory-transportation problem (WITP)* that jointly considers warehouse utilization and capacities, along with inventory and transportation decisions to identify an optimal distribution strategy (see Figure 1). The focus of WITP is to determine the optimal allocation and distribution of products from vendors to stores via warehouses such that total distribution cost is minimized.

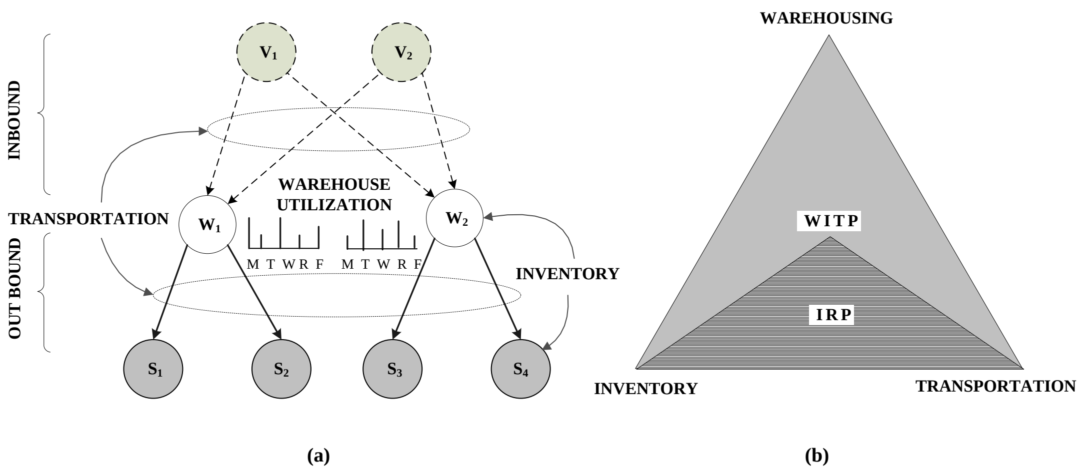

Figure 1: The Warehousing, Inventory, and Transportation Decisions and their Integration in a Multi-Echelon Supply Chain.

The remaining part of this paper is outlined as follows. In Section 2 we briefly review academic literature in this area. In Section 3 we provide details of the WITP and present a cost model for estimating workforce cost at a warehouse. Section 4 presents a mathematical programming model for the WITP. Results based on preliminary experiments are presented in Section 5, followed by a summary in Section 6.

2 Literature Review

Recent years have seen a significant thrust on integrating transportation decisions with inventory in supply chain. The objective has been to trade-off inventory-related and transportation-related costs to minimize supply chain cost. We briefly review integrated models proposed for centralized supply chains.

The presence of a centralized system has led to the questions of when to deliver (timing), how much to deliver (quantity), and how to deliver (mode and routing). From a research standpoint, a popular integrated problem in this area is the *inventory-routing problem (IRP)*, which refers to developing a repeatable distribution strategy that minimizes transportation costs and the number of stock-outs. Both deterministic and stochastic IRP-versions have been introduced in the literature [3, 9]. Abdelmaguid and Dessouky [1] argued that the primary focus of the IRP is on minimizing the total transportation cost, with little consideration for inventory costs. Consequently, they propose an integrated *inventory-distribution problem (IDP)* that considers inventory and transportation costs, allowing backorders, in a multi-period setting. In essence, they suggest that the IRP is a relaxation of the IDP. They present a nonlinear mixed integer programming model for the IDP and solve it using genetic algorithm. They specifically designed the mutation part in the improvement phase of genetic algorithm to investigate partial deliveries, as they can provide significant reductions in transportation and shortage costs.

Lei et al. [10] considered the *production-inventory-distribution-routing problem (PIDRP)*, where the focus is on coordinating the production and transportation schedules between a set of vendors and a set of customers (which could be warehouses). They solve a multi-plant, multi-DC, and multi-period PIDRP using a two-stage sequential approach. Bard and Nananukul [2] solved a one-plant, multi-customer PIDRP assuming a single mode of transportation by employing a reactive tabu search algorithm with path-relinking. Their study differs from the traditional IRP as it considers the trade-off between production decision and inventory level at the facility.

Cetinkaya et al. [4] presented a renewal theoretic model to compute parameters of an integrated inventory-transportation policy where demand follows a general stochastic process. Their research considered one-echelon, one-vendor, one-customer, and one-product scenario, unit transportation cost that includes handling (loading the truck), and inventory related costs at vendor's warehouse. However, they did not capture warehousing costs related to key activities, such as unloading, put-away, picking, and cross-docking in their model.

In the area of warehousing academic literature has focused primarily on warehouse location, design, and operation. White and Francis [15] were probably the first researchers to develop quantitative models to decide between private and leased warehouses. Since then numerous models have been developed to assist in warehouse design, more specifically sizing [6, 8, 11] aisle-layout [7, 14], and operational aspects [12, 13].

From our review of the literature, and industry-practice, we know of no research or tool that integrates warehousing, inventory, and transportation decisions in a single optimization framework. We believe that such integration has the potential of reducing supply chain costs significantly. We now provide details of our proposed research, along with our preliminary work in this area.

3 The Warehouse-Inventory-Transportation Problem

The *warehousing-inventory-transportation problem (WITP)* is to determine the optimal allocation and distribution of product from vendor to stores via one or more warehouses with the objective of minimizing long-term distribution cost. This problem jointly considers warehousing, inventory, and transportation, and addresses the following questions:

- When and in what quantity of each product to order from vendors to replenish warehouses?
- When and what quantities to deliver from warehouses to stores, and which warehouse to source from?
- Is drop-shipping certain products from vendors to stores beneficial?
- Which transportation modes and delivery routes to follow?
- What level of warehouse workforce (permanent and temporary) should be used?

In the WITP we consider the decision of whether or not to *advance* or *delay* shipments depending upon warehouse's workforce utilization, space utilization, and inventory availability. Doing so has cost trade-offs. On one hand, by advancing or delaying shipments warehouse costs may be reduced by better managing the workload on a daily basis, thus reducing variation in workforce utilization. Transportation costs may be reduced due to better consolidation, which may reduce the number of shipments during the time-horizon. However, the stores and warehouses may run a risk of holding too much inventory by advancing or delaying shipments.

The WITP integrates relevant warehousing, inventory, and transportation decisions to tradeoff the associated costs. The warehousing decisions that WITP considers include space, layout, material handling system, workforce planning and scheduling, utilities, and alike. For this study, our focus is on workforce planning.

To model warehouse workforce we use the fact that the workforce level is proportional to the person-hours required for various activities in the warehouse. We consider five key activities; unloading inbound trailers, put-away, picking, loading outbound trailers, and cross-docking. We express the relationship between the required person-hours and the corresponding workforce cost through a piecewise linear cost function; see Figure 2. The parametric curve in the Figure 2 reflects the way most warehouses operate; i.e., most have a mix of permanent and temporary employees, with a possibility of overtime. In the cost function, b_{w1} and b_{w2} represent the levels of permanent and temporary employees, respectively. The region between b_{w2} and b_{w3} represents overtime. We next present our assumptions in developing a mathematical model for the WITP.

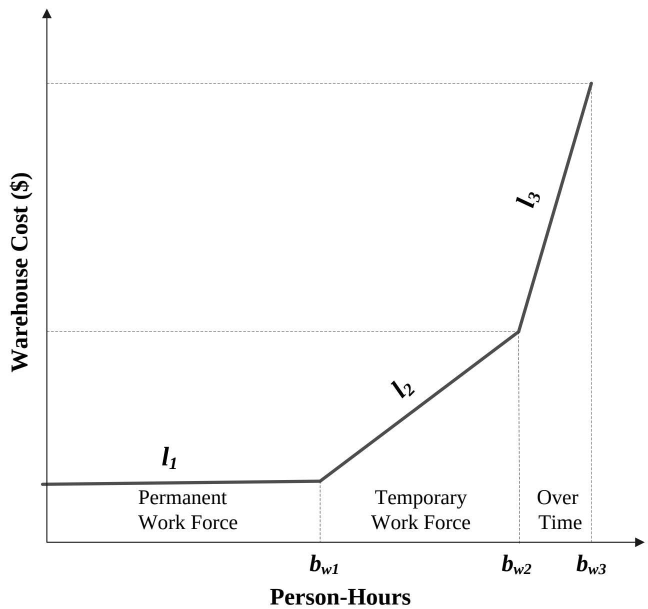

Figure 2: A Piecewise Linear Cost Function to Represent the Relationship Between Required Person-Hours and the Associated Cost.

3.1 Assumptions

We make the following assumptions when developing our mathematical model.

- Vendor has sufficient supplies to meet the demand at warehouses.
- Warehouse utilization is proportional to the utilization of workforce.
- A warehouse can lease space from a third-party logistics provider during the time-horizon.
- Cross-docking is allowed if, in the same time-period, a product inbound from vendor to warehouse could be loaded on a trailer outbound to store to fulfill that store's demand.
- Each warehouse (store) incurs a fixed cost to order a product (in any quantity) from a vendor (warehouse).
- The lead time from vendor to warehouse is one time-period.
- No back orders are allowed.

3.2 A Mixed-Integer Programming Model

Tables 1 and 2 present the parameters and decision variables for the MIP model. The preliminary experiments described in the next section use the values presented in Table 1.

Table 1: Parameters for the MIP Model

Parameter	**Description**	**Value**
w	index for warehouse; $w = 1, 2, \ldots, W$	$W = 1$
s	index for store; $s = 1, 2, \ldots, S$	$S = 2,5$
p	index for product; $p = 1, 2, \ldots, P$	$P = 10, 50, 100$
v	index for vendor; v = 1, 2, …, V	$V = 2$
t	index for time-period; $t = 1, 2, \ldots, T$	$T = 5$
l	index for a piece in the piece-wise linear cost function (see Figure 2); $l = 1, 2, 3$	
Ω_v	set of products p that are sourced from vendor v	
D_{spt}	demand for product p at store s in time-period t	U(0, 100)
V_p	volume of each unit of product p; ft^3	U(0.1, 1)
Q_t	total capacity of each truck in time-period t; ft^3	200
K_w (K_s)	maximum physical space at warehouse w (store s); ft^3	12,500 (2,500)
λ^u (λ^p,λ^k,λ^l,λ^{xd})	rate at which a worker can unload a trailer (put away, pick, load a trailer, cross-dock); units/hr	90 (20, 20, 65, 75)
T^u (T^p,T^k,T^l,T^{xd})	hours during the shift within which unloading (put-away, picking, loading, cross-docking) must be accomplished; hrs/shift	4 (4, 4, 4, 4)
α, β	fraction of workforce used as permanent and for overtime	0.5, 0.2
C_{wp}^h (C_{sp}^h)	holding cost at warehouse w (store s) for product p; \$/unit/time-period	0.05 (0.15)
C_w^a	cost for additional space required at warehouse w during the time-horizon; \$/ft^3	5
C_{w1}^l	loaded cost of a permanent worker ($l = 1$); \$/worker	4
C_{wl}^l	hourly rate for piece $l > 1$ contributing to the warehouse person-hour requirements; \$/hr;	{3, 6}
C_{vwpt}^d (C_{wspt}^d)	fixed (set-up) cost of placing an order to vendor v (warehouse w) from warehouse w (store s) for product p in time-period t; \$/order	10 (5)
C_{vw}^v(C_{ws}^v)	variable volume-based cost of shipment from vendor v (warehouse w) to warehouse w (store s) accounting for the distance between them; \$/ft^3	U(0.004, 0.008) (U(0.008, 0.012))
C_{vw}^f(C_{ws}^f)	fixed cost of a shipment from vendor v (warehouse w) to warehouse w (store s); \$/shipment	10 (10)
M	an arbitrarily large number	10,000

Table 2: Decision Variables in the MIP Model

Decision Variable	Description
x^{i}_{vwpt}	quantity of product *p* *inbound* from vendor *v* to warehouse *w* in time-period *t*
x^{o}_{wspt}	quantity of product *p* *outbound* from warehouse *w* to store *s* in time-period *t*
y_{wpt} (y_{spt})	on-hand inventory of product *p* at warehouse *w* (store *s*) in time-period *t*
n_{vwt} (n_{wst})	number of shipments from vendor *v* (warehouse *w*) to warehouse *w* (store *s*) in time-period *t*
z_{vwpt} (z_{wspt})	1, if an order is placed to vendor *v* (warehouse *w*) from warehouse *w* (store *s*) for product *p* in time-period *t*; 0,otherwise
k^{+}_{w}	additional space required to be leased by warehouse *w* during the time-horizon; ft^3
put_{wpt} ($pick_{wpt}$, xd_{wpt})	number of units of product *p* at warehouse *w* that need to be put-away (picked, cross-docked); units
τ^{u}_{wt} ($\tau^{p}_{wt}, \tau^{k}_{wt}, \tau^{l}_{wt}, \tau^{xd}_{wt}$)	person-hours required at warehouse *w* in time-period *t* for unloading trailers (put-away, picking, loading trailers, and cross-docking)
τ_{wtl}	total person-hours required at warehouse *w* in time-period *t* attributed to piece *l*
b_{wl}	break-points corresponding to the person-hours (permanent, temporary, and overtime) required at warehouse *w* (see Figure 2)

4.1.1 WITP Model

We now present a mixed-integer programming formulation for the WITP.

$$\mathit{Minimize} \quad \sum_{wpt} C^{h}_{wp}\, y_{wpt} + \sum_{spt} C^{h}_{sp}\, y_{spt} + \sum_{vwt} C^{f}_{vw} n_{vwt} + \sum_{wst} C^{f}_{ws} n_{wst} + \sum_{vwpt} C^{v}_{vw} \left(V_p x^{i}_{vwpt}\right) + \sum_{wspt} C^{v}_{ws} \left(V_p x^{o}_{wspt}\right) + \sum_{w} C^{a}_{w}\, k^{+}_{w} + \sum_{vwpt} C^{d}_{vwpt}\, z_{vwpt} + \sum_{wspt} C^{d}_{wspt} z_{wspt} + \sum_{\substack{w \\ l=1}} C^{l}_{wl} b_{wl} + \sum_{\substack{wt \\ l>1}} C^{l}_{wl} \tau_{wtl}$$

Subject to

Store:

$$y_{spt} = y_{sp(t-1)} + \sum_{w} x^{o}_{wspt} - D_{spt} \qquad \forall\, s, p, t \qquad (1)$$

$$\sum_{p} V_p y_{spt} \le K_s \qquad \forall\; s, t \qquad (2)$$

$$Transportation\colon \sum_{p \in \Omega_v} V_p\, x^i_{vwpt} \ \le (0.8\ Q_t) n_{vwt} \qquad \forall\ v, w, t \tag{3}$$

$$\sum_p V_p\, x^o_{wspt} \ \le (0.8\ Q_t) n_{wst} \qquad \forall\ w, s, t \tag{4}$$

$$Ordering\colon \qquad x^i_{vwp(t+1)} \le M z_{vwpt} \qquad \forall\ v, w, p \in \Omega_v, t < T \tag{5}$$

$$x^o_{wsp(t+1)} \le M z_{wspt} \qquad \forall\ w, s, p, t < T \tag{6}$$

$$Warehouse\colon \ \sum_p V_p y_{wpt} \le K_w + k^+_w \qquad \forall\ w, t \tag{7}$$

$$y_{wpt} = y_{wp(t-1)} + put_{wpt} - pick_{wpt} \qquad \forall\ w, p, t \tag{8}$$

$$\sum_v x^i_{vwpt} \quad = xd_{wpt} + put_{wpt} \qquad \forall\ w, p, t \tag{9}$$

$$\sum_s x^o_{wspt} \quad = xd_{wpt} + pick_{wpt} \qquad \forall\ w, p, t \tag{10}$$

$$(T^u \lambda^u) \tau^u_{wt} \ge \sum_{vp} x^i_{vwpt} \qquad \forall\ w, t \tag{11}$$

$$(T^l \lambda^l) \tau^l_{wt} \ge \sum_{sp} x^o_{wspt} \qquad \forall\ w, t \tag{12}$$

$$(T^p \lambda^p) \tau^p_{wt} \ge \sum_p put_{wpt} \qquad \forall\ w, t \tag{13}$$

$$(T^k \lambda^k) \tau^k_{wt} \ge \sum_p pick_{wpt} \qquad \forall\ w, t \tag{14}$$

$$(T^{xd} \lambda^{xd}) \tau^{xd}_{wt} \ge \sum_p xd_{wpt} \qquad \forall\ w, t \tag{15}$$

$$\tau^u_{wt} + \tau^l_{wt} + \tau^p_{wt} + \tau^k_{wt} + \tau^{xd}_{wt} = \ \sum_l \tau_{wtl} \qquad \forall w, t \tag{16}$$

$$0 \ \le\ \tau_{wtl} \ \le\ b_{wl} - b_{w(l-1)} \qquad \forall\ w, t, l > 0 \tag{17}$$

$$b_{w1} \le b_{w2} \le (1 + \alpha) b_{w1} \qquad \forall\ w, l \tag{18}$$

$$b_{w2} \le b_{w3} \le (1 + \alpha + \beta) b_{w1} \qquad \forall\ w, l \tag{19}$$

$$Bounds\colon \qquad x^i_{vwpt}\ , x^o_{wspt}\ , y_{wpt}, y_{spt}, n_{vwt}, n_{wst} \ \in \{0, I^+\} \qquad \forall\ v, w, s, p, t \tag{20}$$

$$z_{vwpt}, z_{wspt} \ \in \{0,1\} \qquad \forall\ v, w, s, p, t \tag{21}$$

$$k^+_w, put_{wpt}, pick_{wpt}, xd_{wpt}, \tau^k_{wt}, \tau^l_{wt}, \tau^p_{wt}, \tau^k_{wt}, \tau^{xd}_{wt}, \tau_{wtl}, b_{wl} \ge 0 \ \ \forall\ w, s, p, t, l \tag{22}$$

The objective of the above model is to minimize the total distribution cost. The cost elements considered include transportation (fixed and variable), holding at warehouse and store, additional warehouse space, and workforce required at the warehouse. Constraints (1) and (2) are related to stores, Constraints (3) and (4) to transportation, Constraints (5) and (6) to order setup, and Constraints (7)-(19) to warehouse space and workforce.

Constraint (1) calculates the on-hand inventory for each product at a store in the current time-period depending on the on-hand inventory in the previous time-period, quantity delivered from warehouses, and the demand at the store. Constraint (2) imposes space constraint at each store.

The transportation capacities (volume-based) for vendor-to-warehouse and warehouse-to-store are modeled through Constraints (3) and (4). Because we do not consider the geometry of the trailer and the products, we restrict the trailer-fill rate to 80% of its volumetric capacity to ensure practical feasibility of loading products in the trailer. Constraints (5) and (6) are used to find if an order is placed by a warehouse (store) to a vendor (warehouse) for a product in a time-period.

Constraint (7) calculates the actual space required at a warehouse allowing for the provision of leasing additional space during the time-horizon. Constraint (8) calculates the on-hand inventory at a warehouse. Constraint (9) balances inbound quantities at the warehouse with cross-docked and put-away quantities, while Constraint (10) balances outbound quantities with cross-docked and picked quantities. The required hours for unloading, put-away, picking, loading, and cross-docking are calculated by Constraints (11)-(15). Constraint (16) calculates the required person-hours at the warehouse to accomplish the five activities during the time-period. Constraint (17) satisfies the incremental person-hours requirement; i.e., first use the permanent workforce, then use temporary, and finally overtime. The requirement that temporary workforce cannot be more than a certain fraction, α, of the permanent workforce at each warehouse is modeled by Constraint (18). Essentially, we are trying to identify the level of permanent and temporary workforce, corresponding to break-points b_{w1} and b_{w2}, respectively, for the time-horizon. Constraint (19) indicates that the allowed overtime at a warehouse is restricted to a certain fraction, β, of the permanent workforce. Constraints (20)-(22) specify bounds on the decision variables.

5 Preliminary Experiments

To evaluate the benefits of the WITP approach, we compare the total distribution cost obtained from the model for WITP to that obtained by sequentially solving the models for ITP (inventory-transportation problem) and WP (warehouse problem). We believe this sequential approach is the current norm in academic literature and industry.

The models for ITP and WP are obtained by decomposing the model for WITP. That is, the model for ITP includes the inventory, transportation, and ordering constraints and associated cost terms in the objective function, while the model for WP includes only the warehousing constraints. Both these models are presented below.

Model for the Inventory-Transportation Problem (ITP)

$$Minimize \quad \sum_{wpt} C^h_{wp}\, y_{wpt} + \sum_{spt} C^h_{sp}\, y_{spt} + \sum_{vwt} C^f_{vw} n_{vwt} + \sum_{wst} C^f_{ws} n_{wst}$$
$$+ \sum_{vwpt} C^v_{vw}\left(V_p x^i_{vwpt}\right) + \sum_{wspt} C^v_{ws}\left(V_p x^o_{wspt}\right) + \sum_{w} C^a_w\, k^+_w$$
$$+ \sum_{vwpt} C^d_{vwpt}\, z_{vwpt} + \sum_{wspt} C^d_{wspt} z_{wspt}$$

Subject to: Constraint-sets (1) – (7) from WITP model

$$y_{wpt} = y_{wp(t-1)} + \sum_{v} x^i_{vwpt} - \sum_{s} x^o_{wspt} \qquad \forall\, w, p, t \qquad (23)$$

$Bounds$:

$$x^i_{vwpt}, x^o_{wspt}, n_{vwt}, n_{wst} \in \{0, I^+\} \qquad \forall\, v, w, s, p, t \qquad (24)$$
$$z_{vwpt}, z_{wspt} \in \{0,1\} \qquad \forall\, v, w, s, p, t \qquad (25)$$
$$k^+_w, y_{wpt}, y_{spt} \geq 0 \qquad \forall\, w, s, p, t \qquad (26)$$

Model for the Warehousing Problem (WP)

$$Minimize \quad \sum_{\substack{w \\ l=1}} C^l_{wl} b_{wl} + \sum_{\substack{wt \\ l>1}} C^l_{wl} \tau_{wtl}$$

Subject to: Constraint-sets (9) – (19) from WITP model

$Bounds$:

$$put_{wpt}, pick_{wpt}, xd_{wpt} \in \{0, I^+\} \qquad \forall w, p, t \qquad (27)$$
$$\tau^k_{wt}, \tau^l_{wt}, \tau^p_{wt}, \tau^k_{wt}, \tau^{xd}_{wt}, \tau_{wtl}, b_{wl} \geq 0 \qquad \forall\, w, t, l \qquad (28)$$

For a given data-set, the optimal solution of ITP provides information about inbound and outbound quantities, warehouse and store inventories, shipments, and ordering. These inbound and outbound quantities, along with warehouse inventory, are used as inputs in the WP model. The optimal solution to the WP provides information about the workforce level at the warehouse, which helps in calculating the warehousing cost. The total distribution cost is then calculated as the sum of inventory, transportation, warehousing, and order set-up costs obtained from both the models.

The total distribution cost resulting from the sequential approach (ITP+WP) is then compared with the optimized solution of integrated WITP model. We also compare the required person-hours for each time-period in the warehouse, and the optimal break-points for all the three types of work forces (permanent, temporary, and over-time) in both the approaches, WITP and ITP+WP. These comparisons are presented in the next section.

5.1 Experimental Set-Up

The optimization models for ITP, WP, and WITP were solved using xPress Optimization software version 12.0. All the computations were performed on a system with 2.53 GHz processor and 512 MB RAM. Several experiments were run with various data-sets to gauge the performance of the solver on these problems. Through initial experiments we observed that though the LP solution was obtained in a few seconds the solver could not obtain optimal solution or prove optimality of the current best solution within 12 hours. Based on these initial experiments, we decided to conduct our preliminary experiments with the following four data-sets:

DS1: v2w1s2p10t5
DS2: v2w1s2p50t5
DS3: v2w1s5p50t5
DS4: v2w1s5p100t5

where v2w1s2p10t5 stands for 2 vendors, 1 warehouse, 2 stores, 10 products, and 5 time-periods.

6 Results and Discussion

The costs of different components (inventory, transportation, warehousing, and order set-up) and the %-savings obtained from the model for WITP, as compared to the sequential ITP+WP approach, are shown in Table 3. A key thing to observe from these results, apart from the 2-6% savings in the total distribution cost, is that the WITP is able to reduce the person-hours at the warehouse in each time-period compared to the ITP+WP approach.

Table 3: Comparison of results obtained from the models for ITP+WP and WITP for four data-sets. (Note: DS = Data-Set, IC = Inventory Cost, TC = Transportation Cost, WC = Warehousing Cost, SC = Order Set-Up Cost, ∑C = Total Cost, WHBP = Warehouse Break-Points)

	ITP + WP						WITP						Savings
DS	IC	TC	WC	SC	∑C	WHBP (hrs)	IC	TC	WC	SC	∑C	WHBP (hrs)	%
	$	$	$	$	$	b1,b2,b3	$	$	$	$	$	b1,b2,b3	
DS1	220	555	359	410	1544	14,21,24	182	555	204	520	1460	8,12,13	5.45
DS2	1042	2929	1805	2075	7852	73,110,125	838	2929	1096	2500	7364	41,61,70	6.21
DS3	1410	7602	2979	5035	17025	118,177,201	1037	7602	2512	5520	16672	96,145,164	2.08
DS4	2583	12680	5982	10310	31555	229,343,389	1844	12680	5063	11260	30846	185,277,314	2.24

For example, for the data-set DS2 (v1w1s2p50t5), we observe a 6.2% of savings in the total cost, accounting mostly due to the differences in the warehousing costs. The model for WITP was able to reduce the warehousing costs from $1,805.27 to $1,095.93, a reduction of nearly 40%. However, the increase in the order set-up cost did reduce these savings quite a bit.

Figures 3-6 represent the differences between the WITP and ITP+WP approaches with respect to the required number of person-hours in each time-period. From Figure 3 we observe that, for the ITP+WP approach, in time-periods 1 and 4, the number of required person-hours is relatively high requiring overtime to accomplish the workload during that time-period. However, during time-periods 2 and 5 the workload was relatively low resulting in no need for overtime hours; in fact, no temporary workers are required during time-period 5. Such a large variation in the amount of workload across time-periods in a time-horizon is commonly experienced by many warehouses, and makes it relatively difficult for the warehouse manager to plan the workforce.

With the integrated WITP approach, not only the required total person-hours are reduced considerably, but also the person-hours are well balanced across all time-periods. Both these aspects make it easy for the warehouse manager to efficiently manage the workforce in the warehouse. This effect is observed for the remaining three data-sets, as illustrated in Figures 4-6.

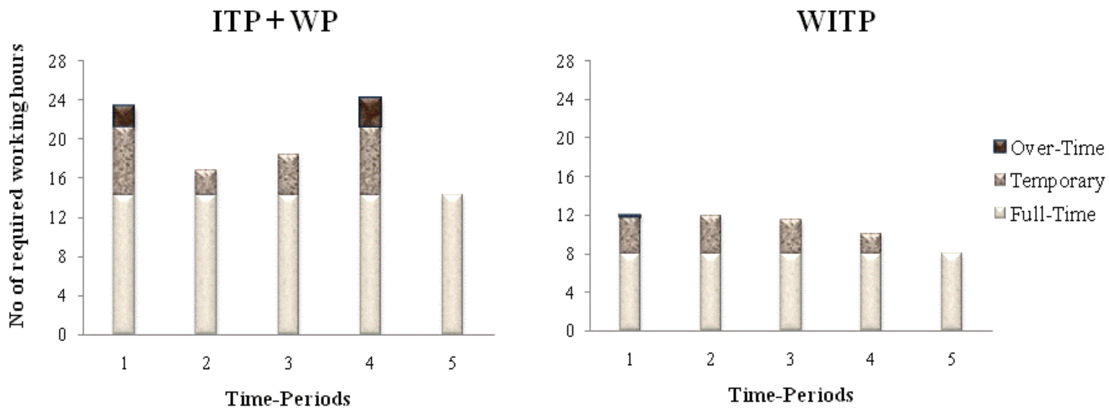

Figure 3: Daily requirement of person-hours at the warehouse obtained through the ITP+WP and WITP approaches for data-set DS1.

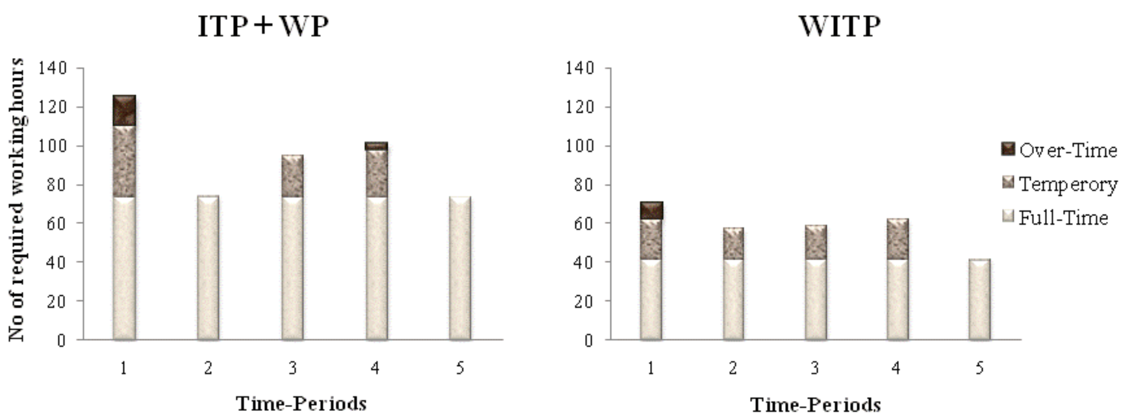

Figure 4: Daily requirement of person-hours at the warehouse obtained through the ITP+WP and WITP approaches for data-set DS2.

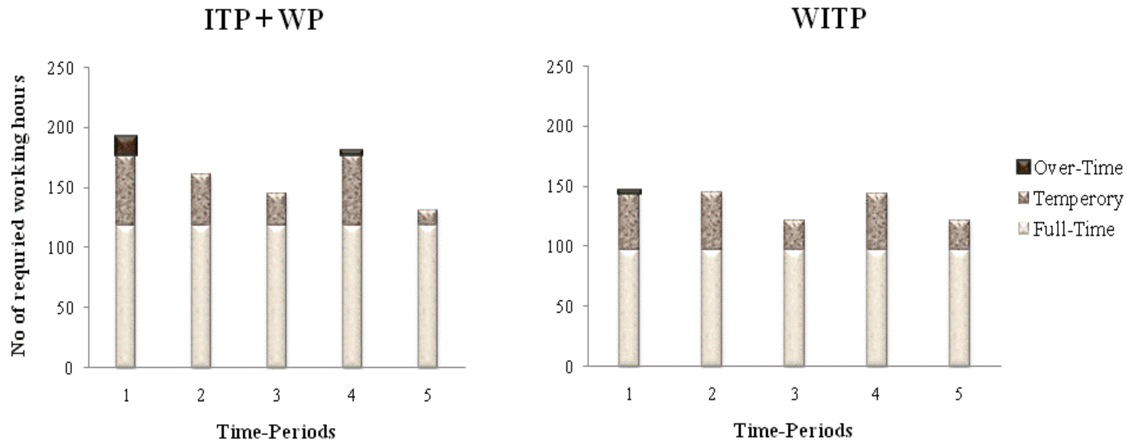

Figure 5: Daily requirement of person-hours at the warehouse obtained through the ITP+WP and WITP approaches for data-set DS3.

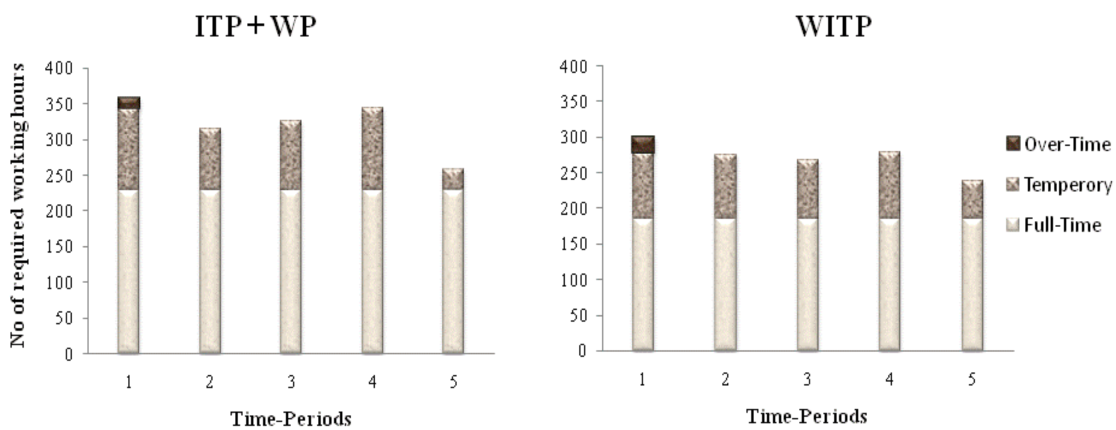

Figure 6: Daily requirement of person-hours at the warehouse obtained through the ITP+WP and WITP approaches for data-set DS4.

7 Summary

In this paper we introduced the integrated warehousing-inventory-transportation problem (WITP). The WITP was motivated from our observations of an apparel supply chain in which warehousing decisions succeeded transportation and inventory decisions. Consequently, warehouses have operated in a reactive mode, which has led to large variations in the workforce utilization, thus affecting the supply chain’s bottom-line.

The WITP trades off warehousing, inventory, and transportation decisions such that the long-run distribution cost is minimized. Aspects such as warehouse workforce (permanent, temporary, and over-time) and space to accomplish major warehousing activities such as unloading and loading a trailer, put-away, picking, and cross-docking were considered. Preliminary experiments suggest that our proposed model for WITP was able to reduce the mean and variance in number of person-hours at the warehouse. A savings in the range of 2-6% in total distribution cost was also observed.

Our current efforts are focused on developing a heuristic algorithm to solve realistic problem-sizes. As many supply chains prefer a policy-based distribution strategy, we intend to identify easy-to-implement and repeatable strategies that ensure near-optimal solution to the WITP.

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