# THE EFFECT OF INTERACTION BETWEEN THE PRODUCTION SYSTEM (PS) AND A LOOPED CONVEYOR-BASED MATERIAL HANDLING SYSTEM (LCMHS) IN A MANUFACTURING FACILITY

**Vernet Lasrado**
**University of Central Florida**

**Dima Nazzal**
**University of Central Florida**

## Abstract

In this paper, we provide empirical evidence that shows the effect of the interaction between the production system (PS) and a looped conveyor-based material handling system (LCMHS) in a manufacturing facility. A rudimentary simulation model captures the interaction between the two systems. Varying several key factors, we test for a statistically significant difference in the work in process (WIP) of the production system with and without the LCMHS to find if the squared coefficient of variation (SCV) of the interarrival time distribution to the PS is affected. The results suggest the need to model the interaction between the two systems in order to obtain a more representative estimate of the WIP in a manufacturing facility.

## 1  Introduction

When analyzing a general production or manufacturing facility, two systems are usually considered: the production system (PS) and the material handling system (MHS). When these systems are modeled independently, as is often the case in the literature, the interaction between such systems and their impact upon one another can be easily overlooked.

Queuing theory is used as a foundation for the analytical modeling of the PS of the manufacturing facility. For a thorough review of existing applications of queueing theory to manufacturing facilities see Govil and Fu [1]. A popular method is the parametric-decomposition method referred to as the queuing network analyzer (QNA) [2-5]. The PS is a network of stations (i=1,2,…,M) broken into nodes, with each represented as a G/G/m queue. The arrival to each queue is assumed to be a renewal process [6]. For each station $i$, the interarrival time distribution and service time distribution are each described by their first two moments, viz., the mean time and the squared coefficient of variation (SCV). Equation (1) estimates the WIP for a G/G/m queue [7] at station $i$:

$$L_i = \left( \frac{c_{ai}^2 + c_{si}^2}{2} \right) \left[ \frac{(\rho_i)^{\left( \sqrt{2(m_i+1)}+1 \right)}}{1-\rho_i} \right] + \rho_i \,. \tag{1}$$

Hseih and Bozer [8] present an analytical model of a LCMHS. Let $C\lambda_i$ be the flow rate of jobs from station $i$ and $C\lambda_{ij}$ the flow rate of jobs from station $i$ to station $j$ in the PS where $C > 0$. $C$ is used as a factor to allow uniform variations in the flow rates. Each station $i$ has a loading and unloading station to/from the LCMHS. The unloading station is followed by a loading station in the direction of the LCMHS's flow [9]. A segment of the LCMHS is defined as the area of the conveyor between loading (unloading) and unloading (loading) stations. Let $\alpha_i$ be the average number of loads per time unit on segment $i$ (see [8] for details on deriving $\alpha_i$). The utilization of the conveyor is given by:

$$\rho_{Conv} = \max_i \left[ (C\lambda_i + \alpha_i)/v \right]. \tag{2}$$

where, the stability condition of the conveyor is $\rho_{Conv} < 1$. Here, $v$ is the speed of the conveyor in windows per time unit, where a window is the length of a conveyor defined to hold at most one job.

A limited body of work ([10],[11]) attempts to estimate the effect of the MHS on the WIP at the PS of the manufacturing facility by capturing the interaction between both systems, i.e. the effect of the MHS on the variability of the interarrival time distribution to the machines in the PS. Benjaffar [10] studies the effect of the interaction between a vehicle-based MHS and PS by modeling the effect of variability of the transit time distribution of the vehicle-based MHS on the variability of the interarrival time distribution between the different stations in the PS. While Benjaffar [10] focuses on vehicle-based MHS, the proposed research will model a PS served with a closed-loop conveyor as the MHS.

All factors being constant, the WIP in a PS differs from the WIP as described in equation (1) due to the effect of variability of the transit time distribution of the MHS on the variability of the interarrival times to the different stations in the PS. Specifically, if the MHS increases (decreases) the SCV of the interarrival times to the different stations in the PS, the WIP in the PS increases (decreases).

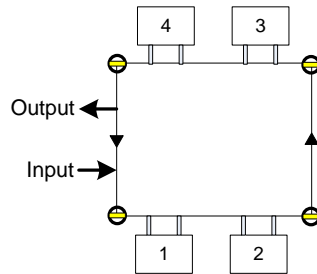To further clarify this effect, consider a simple PS with a MHS as illustrated below:



Figure 1: Illustrative Manufacturing Facility

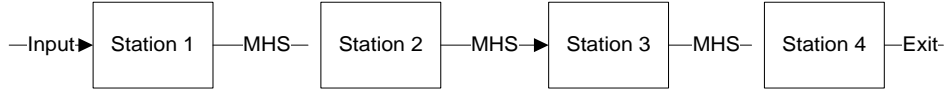Assume a load takes the following route through the facility:



Figure 2: Flowchart of Queueing Network

The described effect of the MHS on the interarrival time distribution of the load traveling from Station 1 to Station 2 of the PS can be characterized as:

$$
\begin{aligned}
c_{d_1 \sim QNA}^2 &= c_{a_1 \sim QNA}^2 \cdot \left(1 - \rho_{1 \sim QNA}^2\right) + c_{s_1 \sim QNA}^2 \cdot \rho_{1 \sim QNA}^2 = c_{a_1 \sim MHS}^2 \\
c_{d_1 \sim MHS}^2 &= c_{a_1 \sim MHS}^2 \cdot \left(1 - \rho_{1 \sim MHS}^2\right) + c_{s_1 \sim MHS}^2 \cdot \rho_{1 \sim MHS}^2 = c_{a_2 \sim QNA}^2
\end{aligned}
\tag{3}
$$

If the MHS causes the SCV of the interarrival times of loads traveling from Station 1 to Station 2 to decrease (increase), as per equation (1) and equation (3), the WIP in the PS is reduced (increased). Hence, measuring the effect of the MHS on the WIP at Station $i$ in the PS is the same as measuring the effect of the MHS on $c_{ai}^2$ to Station $i$ in the PS.

Section 2 describes the experiment in detail; Section 3 describes the design and parameters of the experiment; Section 4 discusses the results; and Section 5 presents the conclusions and suggestions for further research.

## 2  Description of the Experiment

The purpose of this experiment is to verify the effect of the MHS on the variability of the interarrival time distribution. Figure 3 shows a 24-bay facility which is a simplified depiction of a 300mm wafer fabrication facility as described in [12] and [13]. The manufacturing facility can be decomposed into two interacting subsystems: a PS and a LCMHS.

Each bay has a loading and an unloading station. Obviously, the loading station is where a load enters the conveyor (loads are created) and the unloading station is where all loads depart from the conveyor (loads are destroyed). We assume all queues have infinite capacity. We choose this design because it greatly simplifies and expedites the simulation, while allowing us to track the interarrival time distribution to the PS.
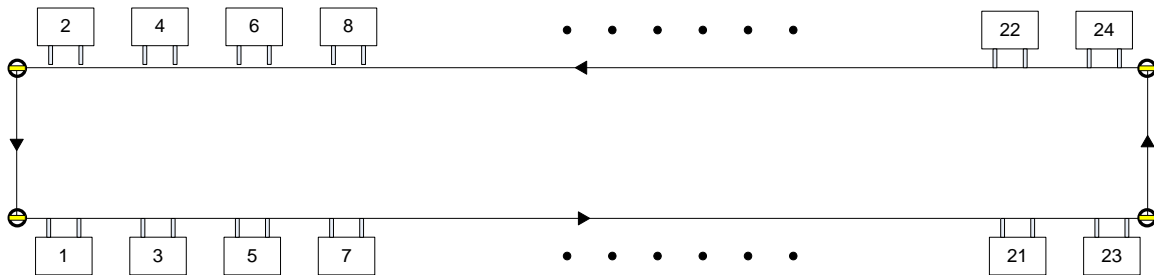


Figure 3: 300mm Wafer Fabrication Facility

We simulate two distinct layouts: Layout A and Uniform Layout, using Automod simulation software from Brooks Automation. The flow rates (in hours) for Layout A are represented in the

"from-to" matrix as shown in Table A.1. In the Uniform Layout all bays send the same number of loads to all other bays; the elements in the from-to matrix of the flow rates are:

$$\lambda_{ij} = \begin{cases} 0 & \forall\ i = j \\ 0.5273 & \forall\ i \neq j \end{cases}$$

## 3 Design of the Experiment

Assume there is no conveyor and all flow rates between the bays are external arrivals to the individual bays. For example, if bay 1 has arrivals from bay 3 and bay 5 in the from-to matrix, then both arrivals are considered external arrivals to bay 1. If the LCMHS has an effect on the interarrival time distribution, the $c_{ai}^2$ of the external arrival distribution as recorded in the simulation will differ from the $c_{ai}^2$ of the theoretical distribution. For illustrative purposes, the arrivals in the simulation follow a Poisson Process. Hence, the external arrivals to any station $i$ of the PS should have $c_{ai}^2$ equal to 1. If the LCMHS has no statistically significant effect on the arrivals to the PS, $c_{ai}^2$ as recorded in the experiment will be equal to 1, i.e. the same as loads arriving according to a Poisson process. The experiment is validated if a statistically significant difference exists between the expected $c_{ai}^2$ and the experimental $c_{ai}^2$.

### 3.1 Experiment

Based on empirical observances, we note that any combination of conveyor speed and flow rates that yields the same utilization will have statistically indifferent results. Hence, we test our two layouts at a variety of conveyor utilization levels as shown in Table 1. The utilization levels are determined as described by equation (2). Each simulation is warmed up for 2 days and then run for 5 replications of 30 days. The SCV of the interarrival time distributions is captured for each replication.

Table 1: Test Levels at Different Conveyor Utilizations

| Rho | 0.1500 | 0.3000 | 0.4500 | 0.6000 | 0.7500 | 0.9000 |
|---|---|---|---|---|---|---|
| Speed | 3 | 3 | 3 | 3 | 3 | 3 |
| C Uniform Layout | 11.132 | 22.264 | 33.396 | 44.528 | 55.66 | 66.792 |
| C Layout A | 10.641 | 21.282 | 31.923 | 42.564 | 53.205 | 63.846 |

### 3.2 Hypothesis Testing

Let $\Delta$ be the average of the absolute differences between 1 and the $c_{ai}^2$ as obtained in the simulation (since the SCV for the exponential interarrival time is 1 as prescribed by the Poisson Process)

$$\Delta = \frac{\sum_{i=1}^{24} \left| c_{ai}^2 - 1 \right|}{24} \tag{4}$$

If $\Delta$ statistically differs from 0, it implies that the conveyor has altered the variability of interarrival time distribution. We run a standard t-test with a null hypothesis that tests $\Delta$=0 with α=0.05.

## 4 Results

The aggregated results for the interarrival time distribution for Layout A appear in Table 2 and the results for the Uniform Layout appear in Table 3. See Tables B.1 and B.2 for the raw output from the simulations of Layout A and the Uniform Layout.

Table 2: SCV of Interarrival Time Descriptive Statistics for Layout A

| Rho | Average | Δ | StDev (Δ) | SE (Δ) | 95% CI (Δ) | T (Δ) | P (Δ) |
|------|---------|--------|-----------|--------|------------------------|-------|-------|
| 0.15 | 0.9993 | 0.0067 | 0.0063 | 0.0013 | ( 0.00394,  0.00937) | 5.08 | 0.00 |
| 0.30 | 0.9995 | 0.0051 | 0.0038 | 0.0008 | (0.003440, 0.006740) | 6.40 | 0.00 |
| 0.45 | 0.9977 | 0.0050 | 0.0037 | 0.0008 | (0.003412, 0.006588) | 6.53 | 0.00 |
| 0.60 | 0.9943 | 0.0091 | 0.0083 | 0.0017 | ( 0.00550,  0.01266) | 5.26 | 0.00 |
| 0.75 | 0.9886 | 0.0179 | 0.0161 | 0.0034 | ( 0.01093,  0.02481) | 5.34 | 0.00 |
| 0.90 | 0.9685 | 0.0350 | 0.0312 | 0.0065 | ( 0.02148,  0.04847) | 5.38 | 0.00 |
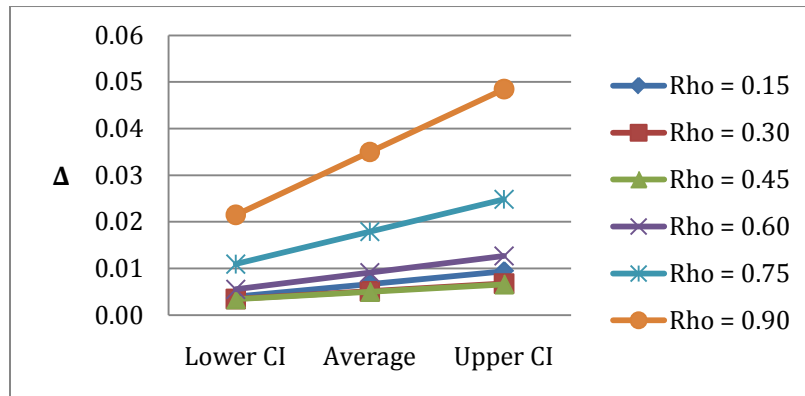


Figure 4: 95% confidence interval of Δ for Layout A

For Layout A, we reject the null hypothesis at all utilization levels, since the p-value is less than 0.05 and the lower bound of the 95% confidence interval of $\Delta$ is above zero as in Table 2 and Figure 4. As can be seen, the absolute deviation from 1 increases as the utilization of the LCMHS increases. Interestingly, Figure 6 shows that the average $c_{ai}^2$ decreases as the utilization of the conveyor increases.

Table 3: SCV of Interarrival Time Descriptive Statistics for Uniform Layout

| Rho | Average | Δ | StDev (Δ) | SE (Δ) | 95% CI (Δ) | T (Δ) | P (Δ) |
|------|---------|--------|-----------|--------|------------------------|--------|-------|
| 0.15 | 0.9992 | 0.0054 | 0.0047 | 0.0010 | (0.003447, 0.007438) | 5.64 | 0.00 |
| 0.30 | 0.9990 | 0.0026 | 0.0025 | 0.0005 | (0.001579, 0.003683) | 5.18 | 0.00 |
| 0.45 | 0.9943 | 0.0059 | 0.0025 | 0.0005 | (0.004828, 0.006936) | 11.54 | 0.00 |
| 0.60 | 0.9829 | 0.0171 | 0.0036 | 0.0007 | (0.015559, 0.018575) | 23.41 | 0.00 |
| 0.75 | 0.9590 | 0.0410 | 0.0028 | 0.0006 | (0.039844, 0.042177) | 72.75 | 0.00 |
| 0.90 | 0.9291 | 0.0709 | 0.0026 | 0.0005 | (0.069853, 0.072037) | 134.39 | 0.00 |
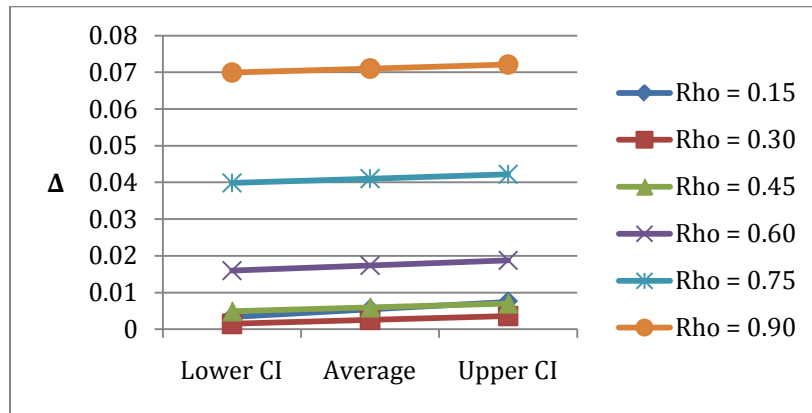


Figure 5: 95% confidence interval of Δ for Uniform Layout

For the Uniform Layout, we reject the null hypothesis at all utilization levels, since the p-value is less than 0.05 and the lower bound of the 95% confidence interval for Δ is above zero as in Table 3 and Figure 5. As can be seen, the absolute deviation from 1 increases as the utilization of the LCMHS increases. Again, Figure 6 shows that the average $c_{ai}^2$ decreases as the utilization of the conveyor increases.



Figure 6: Aggregate SCV of Layout A versus Uniform Layout

# 5  Conclusions and Future Research

This research has presented empirical evidence that demonstrates the effect of the LCMHS on the WIP in the PS. As discussed in Section 4, definitive evidence shows the effect of the LCMHS on the WIP in the PS.

As the utilization of the LCMHS increases, the variability of the interarrival times to the PS decreases. As a result of the decreased variability in the PS (equation 1), it is clear that the WIP in the system decreases as the utilization of the LCMHS in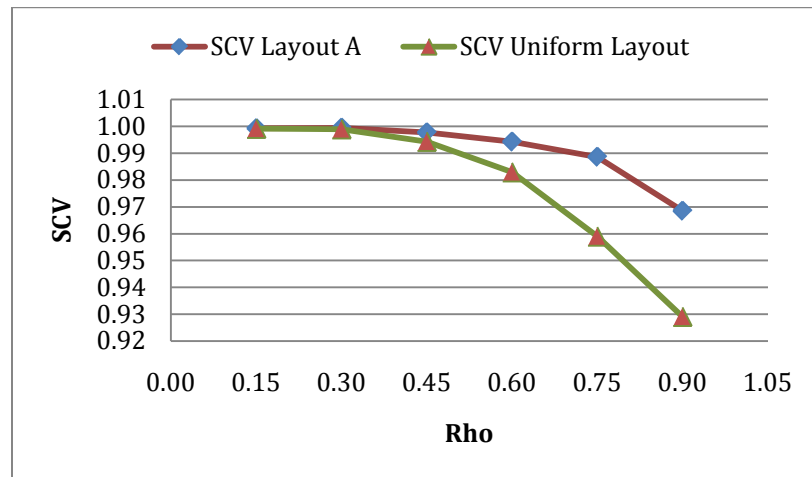creases. This result is counter-intuitive, but can be explained by the fact that as the utilization of the conveyor increases, the availability of open windows decreases. Hence, the arrival of the loads between bays is streamlined.

We conclude, therefore, that in very busy systems, a LCMHS will reduce the overall variability of the system, thereby effectively reducing the system's WIP.

We are now developing an analytical model to estimate the SCV of departures from the conveyor to the stations in order to estimate the overall WIP in the PS and MHS. We intend to employ this analytical model in an optimization framework to design the layout of the machines in order to minimize total WIP. We believe the model will be superior to the traditional method of designing layouts to minimize the travel distances.

# Appendix A

Table A.1: $\lambda_{ij}$ Flow Rate Matrix / From-To Matrix

| From/To | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 7.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 | 0.92 | 0.00 | 0.73 | 0.00 | 0.00 | 1.12 | 0.25 | 0.34 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.60 | 3.63 | 0.64 | 0.62 |
| 3 | 1.44 | 1.01 | 0.00 | 1.06 | 1.29 | 0.26 | 0.51 | 1.46 | 1.36 | 0.48 | 0.00 | 0.00 | 0.22 | 0.03 | 0.06 | 0.00 | 0.46 | 0.36 | 0.00 | 1.28 | 0.41 | 0.49 | 0.27 | 1.27 |
| 4 | 2.29 | 0.00 | 1.16 | 0.00 | 1.15 | 0.35 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 | 0.17 | 0.00 | 0.21 | 0.00 | 0.00 | 1.94 | 0.05 | 0.00 | 0.00 | 2.16 |
| 5 | 1.45 | 0.57 | 0.64 | 0.63 | 0.00 | 0.23 | 0.47 | 3.54 | 0.32 | 1.08 | 0.00 | 0.00 | 0.23 | 0.10 | 0.07 | 0.00 | 0.42 | 0.37 | 0.00 | 0.89 | 0.63 | 0.89 | 0.29 | 0.68 |
| 6 | 0.00 | 0.00 | 0.52 | 0.49 | 0.14 | 0.00 | 0.63 | 0.00 | 0.00 | 0.81 | 0.00 | 0.00 | 1.17 | 0.00 | 0.41 | 0.00 | 0.00 | 0.46 | 1.24 | 0.79 | 0.00 | 0.00 | 0.00 | 0.46 |
| 7 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 10.64 | 0.00 | 0.00 | 0.00 | 0.00 |
| 8 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 4.08 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.91 | 3.19 | 0.00 | 0.00 |
| 9 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.02 | 4.92 | 0.00 | 0.00 |
| 10 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.55 | 2.48 | 1.74 | 0.00 | 0.00 |
| 11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.01 | 4.93 | 0.00 | 0.00 |
| 12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 13 | 0.00 | 2.73 | 2.47 | 1.07 | 2.99 | 0.95 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 | 0.00 | 0.00 | 0.05 | 0.13 | 0.00 | 0.00 | 1.35 | 2.12 | 0.31 | 0.09 | 0.21 | 0.04 | 0.44 |
| 14 | 0.00 | 0.49 | 1.60 | 0.00 | 0.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 | 0.00 | 0.00 | 0.17 | 0.11 | 0.00 | 0.00 | 0.63 | 1.26 | 4.51 | 0.27 |
| 15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.68 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.32 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 1.39 |
| 16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 6.61 |
| 17 | 0.00 | 0.00 | 2.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 5.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 18 | 0.00 | 0.00 | 0.55 | 1.04 | 0.34 | 1.78 | 0.00 | 0.00 | 0.00 | 0.13 | 7.60 | 0.00 | 0.65 | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 | 0.60 | 0.14 | 1.65 | 0.00 | 0.00 | 1.56 |
| 19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 4.83 | 0.00 | 0.00 | 0.00 | 0.00 | 1.75 | 0.00 | 0.00 | 3.52 | 0.00 | 0.99 | 1.85 |
| 20 | 2.10 | 0.00 | 0.49 | 3.65 | 1.34 | 0.34 | 2.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 | 1.52 | 0.00 | 0.26 | 2.65 | 7.90 | 0.00 | 0.06 | 0.00 | 0.00 | 0.84 |
| 21 | 0.64 | 0.58 | 0.23 | 0.17 | 0.44 | 0.42 | 0.20 | 1.69 | 0.28 | 0.51 | 1.65 | 0.00 | 0.70 | 3.05 | 0.94 | 2.47 | 0.44 | 3.86 | 1.36 | 0.35 | 0.00 | 2.07 | 0.86 | 0.44 |
| 22 | 0.00 | 1.84 | 0.47 | 0.00 | 1.11 | 0.00 | 0.00 | 2.35 | 0.40 | 0.72 | 0.00 | 0.00 | 0.00 | 4.98 | 0.00 | 4.15 | 1.03 | 4.73 | 0.00 | 0.00 | 2.39 | 0.00 | 1.11 | 0.02 |
| 23 | 0.00 | 2.90 | 0.19 | 0.00 | 1.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.58 | 0.00 | 0.00 | 0.15 | 0.09 | 0.00 | 1.35 | 0.40 | 0.66 | 0.00 | 0.45 |
| 24 | 0.00 | 0.40 | 1.87 | 2.39 | 1.55 | 0.46 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 | 0.00 | 1.13 | 0.18 | 0.06 | 0.00 | 4.86 | 0.32 | 0.67 | 1.86 | 1.52 | 1.28 | 0.38 | 0.00 |

# Appendix B

Table B.1: Δ of Interarrival Time Distribution for Layout A

| Station\Rho | 0.1500 | 0.3000 | 0.4500 | 0.6000 | 0.7500 | 0.9000 |
|---|---|---|---|---|---|---|
| 1 | 0.0036 | 0.0033 | 0.0027 | 0.0007 | 0.0026 | 0.0183 |
| 2 | 0.0018 | 0.0063 | 0.0004 | 0.0051 | 0.0135 | 0.0573 |
| 3 | 0.0021 | 0.0038 | 0.0070 | 0.0028 | 0.0006 | 0.0184 |
| 4 | 0.0132 | 0.0050 | 0.0020 | 0.0114 | 0.0185 | 0.0387 |
| 5 | 0.0068 | 0.0105 | 0.0096 | 0.0005 | 0.0234 | 0.0198 |
| 6 | 0.0002 | 0.0002 | 0.0052 | 0.0088 | 0.0302 | 0.0686 |
| 7 | 0.0036 | 0.0038 | 0.0013 | 0.0036 | 0.0103 | 0.0279 |
| 8 | 0.0143 | 0.0137 | 0.0062 | 0.0113 | 0.0087 | 0.0031 |
| 9 | 0.0187 | 0.0140 | 0.0075 | 0.0113 | 0.0080 | 0.0015 |
| 10 | 0.0066 | 0.0022 | 0.0017 | 0.0025 | 0.0004 | 0.0000 |
| 11 | 0.0047 | 0.0071 | 0.0019 | 0.0044 | 0.0095 | 0.0275 |
| 12 | 0.0046 | 0.0021 | 0.0040 | 0.0150 | 0.0293 | 0.0676 |
| 13 | 0.0188 | 0.0047 | 0.0014 | 0.0077 | 0.0015 | 0.0031 |
| 14 | 0.0043 | 0.0018 | 0.0043 | 0.0053 | 0.0244 | 0.0203 |
| 15 | 0.0044 | 0.0068 | 0.0092 | 0.0114 | 0.0145 | 0.0094 |
| 16 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 17 | 0.0009 | 0.0010 | 0.0037 | 0.0046 | 0.0038 | 0.0280 |
| 18 | 0.0111 | 0.0009 | 0.0033 | 0.0020 | 0.0131 | 0.0201 |
| 19 | 0.0097 | 0.0028 | 0.0006 | 0.0020 | 0.0009 | 0.0047 |
| 20 | 0.0193 | 0.0024 | 0.0099 | 0.0265 | 0.0349 | 0.0601 |
| 21 | 0.0005 | 0.0030 | 0.0075 | 0.0169 | 0.0395 | 0.0992 |
| 22 | 0.0009 | 0.0065 | 0.0026 | 0.0068 | 0.0211 | 0.0401 |
| 23 | 0.0025 | 0.0094 | 0.0144 | 0.0341 | 0.0649 | 0.1130 |
| 24 | 0.0004 | 0.0056 | 0.0085 | 0.0141 | 0.0373 | 0.0580 |

Table B.2: Δ of Interarrival Time Distribution for Uniform Layout

| Station\Rho | 0.1500 | 0.3000 | 0.4500 | 0.6000 | 0.7500 | 0.9000 |
|---|---|---|---|---|---|---|
| 1 | 0.0090 | 0.0002 | 0.0022 | 0.0169 | 0.0446 | 0.0743 |
| 2 | 0.0075 | 0.0018 | 0.0044 | 0.0191 | 0.0388 | 0.0727 |
| 3 | 0.0206 | 0.0005 | 0.0040 | 0.0208 | 0.0442 | 0.0724 |
| 4 | 0.0088 | 0.0004 | 0.0078 | 0.0187 | 0.0419 | 0.0729 |
| 5 | 0.0025 | 0.0032 | 0.0061 | 0.0166 | 0.0436 | 0.0678 |
| 6 | 0.0113 | 0.0016 | 0.0069 | 0.0140 | 0.0470 | 0.0717 |
| 7 | 0.0051 | 0.0024 | 0.0041 | 0.0119 | 0.0400 | 0.0684 |
| 8 | 0.0013 | 0.0068 | 0.0055 | 0.0214 | 0.0403 | 0.0675 |
| 9 | 0.0083 | 0.0024 | 0.0051 | 0.0146 | 0.0361 | 0.0713 |
| 10 | 0.0038 | 0.0021 | 0.0098 | 0.0138 | 0.0379 | 0.0678 |
| 11 | 0.0012 | 0.0003 | 0.0028 | 0.0126 | 0.0393 | 0.0649 |
| 12 | 0.0006 | 0.0001 | 0.0034 | 0.0209 | 0.0416 | 0.0727 |
| 13 | 0.0030 | 0.0002 | 0.0080 | 0.0202 | 0.0395 | 0.0706 |
| 14 | 0.0116 | 0.0042 | 0.0022 | 0.0125 | 0.0379 | 0.0730 |
| 15 | 0.0033 | 0.0103 | 0.0063 | 0.0231 | 0.0463 | 0.0741 |
| 16 | 0.0059 | 0.0050 | 0.0038 | 0.0097 | 0.0405 | 0.0689 |

| 17 | 0.0068 | 0.0035 | 0.0050 | 0.0189 | 0.0407 | 0.0741 |
| 18 | 0.0082 | 0.0053 | 0.0107 | 0.0182 | 0.0416 | 0.0703 |
| 19 | 0.0041 | 0.0007 | 0.0082 | 0.0166 | 0.0411 | 0.0725 |
| 20 | 0.0011 | 0.0050 | 0.0108 | 0.0182 | 0.0379 | 0.0714 |
| 21 | 0.0026 | 0.0028 | 0.0065 | 0.0142 | 0.0407 | 0.0738 |
| 22 | 0.0033 | 0.0026 | 0.0064 | 0.0195 | 0.0378 | 0.0713 |
| 23 | 0.0005 | 0.0007 | 0.0076 | 0.0219 | 0.0419 | 0.0713 |
| 24 | 0.0003 | 0.0013 | 0.0037 | 0.0152 | 0.0431 | 0.0670 |

## References

[1] Govil, M., & Fu, M., "Queueing theory in manufacturing: A survey," *Journal of manufacturing systems*, 18, 3, 214-240 (1999).

[2] Whitt, W., "The queueing network analyzer," *Bell System Technical Journal*, 62, 9, 2779-2815 (1983).

[3] Whitt, W., "Performance on the queueing network analyzer," *Bell System Technical Journal*, 62, 9, 2817-2843 (1983).

[4] Segal, M., & Whitt, W., "A queueing network analyzer for manufacturing," *Proceeding of the 12th International Teletraffic Congress*, International Teletraffic Congress, , 1146-1152 (1989).

[5] Meng, G., & Heragu, S., "Batch size modeling in a multi-item, discrete manufacturing system via an open queuing network," *IIE transactions*, 36, 8, 743-753 (2004).

[6] Pyke, R., "Markov renewal processes: definitions and preliminary properties," *The Annals of Mathematical Statistics*, 32, 4, 1231-1242 (1961).

[7] Hopp, W., & Spearman, M., *Factory physics: foundations of manufacturing management*, McGraw-Hill/Irwin, Columbus, (2000).

[8] Hsieh, Y., & Bozer, Y., "Analytical Modeling of Closed-Loop Conveyors with Load Recirculation," *Computational Science and Its Applications – ICCSA 2005*, Springer, New York, 437-447 (2005).

[9] Bozer, Y., & Hsieh, Y., "Expected waiting times at loading stations in discrete-space closed-loop conveyors," *European Journal of Operational Research*, 155, 2, 516-532 (2004).

[10] Benjaafar, S., "Design of plant layouts with queueing effects," *Management Science*, 48, 5, 679-704 (2002).

[11] Raman, D., Nagalingam, S., & Gurd, B., "A genetic algorithm and queuing theory based methodology for facilities layout problem," *International Journal of Production Research*, 47, 20, 1-25 (2008).

[12] Nazzal, D., & El-Nashar, A., "Survey of research in modeling conveyor-based automated material handling systems in wafer fabs," *Proceedings of the 39th Conference on Winter Simulation*, Winter Simulation Conference, , 1781-1788 (2007).

[13] SEMATECH, Fab simulation modeling software. Available online: http://ismi.sematech.org/modeling/simulation/index.htm#models