MONITORING INVENTORY ACCURACY WITH STATISTICAL PROCESS CONTROL

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Abstract

Inventory accuracy is critical in most industrial environments such as distribution, warehousing, and retail. Many companies use a technique called cycle counting and have realized outstanding results in monitoring and improving inventory accuracy. The time and resources to complete cycle counting are sometimes limited or not available. In this work, we promote statistical process control (SPC) to monitor inventory accuracy. Specifically, we model the complex underlying environments with mixture distributions to demonstrate sampling from a mixed but stationary process. For our particular application, we concern ourselves with data that result from inventory adjustments at the stock keeping unit (SKU) level when a given SKU is found to be inaccurate. We provide estimates of both the Type I and Type II errors when a classic C chart is used. In these estimations, we use both analytical as well as simulation results, and the findings demonstrate the environments that might be conducive for SPC approach.

1. Introduction

Inventory record accuracy is vital to any company with high levels of inventory. Failure to keep accurate inventory records can result in loss of product, time wasted correcting records, product not in stock for consumers, and overstock of items. Inventory accuracy is realized when the actual on hand inventory equals recorded inventory. This has become a challenging task for some environments (e.g. large retail stores, distribution centers,

etc.) because they often have thousands of different stock keeping units (SKU's) in their inventory.

Cycle counting is currently the most common and established method used by companies to keep inventory record accuracy. Cycle counting has generally replaced annual physical inventory checks. Cycle counting has been accepted as a better method as it doesn't require the entire environment to shut down to count SKU's. Physical inventory checks are not only tedious and stressful, and they can result in errors due to the time constraints on counting the SKU's. Cycle counting counts subsets of inventory to check that the actual on hand inventory equals the recorded inventory. If there are differences between the two, the errors are corrected. Cycle counting has been found to be less disruptive to daily operations, provides ongoing measure of inventory accuracy, and can be molded to focus on items with higher value.

It is believed that with the correct execution of cycle counting that any company can have 95% or better accuracy. The dilemma for a large company is that it takes a large amount of resources, labor hours, and money to ensure that cycle counting is implemented correctly. Comprehensively for large environments there is a need for a method to keep high levels of inventory accuracy while minimizing the use of available resources. As companies strive to be more efficient, the cost competitive pressures mount on the effective use of resources.

Statistical Process Control (SPC) is a proven statistical method used to monitor processes and improve quality using variance reduction. SPC utilizes random samples to monitor and control a process to ensure it is operating correctly and producing conforming parts. In our inventory accuracy domain there is an opportunity to utilize random samples rather than the comprehensive approach of cycle counting, but statistically valid approaches must be utilized as control type I and type II errors. As such statistical process control is an ideal application for monitoring inventory accuracy.

The two methods that can be used are a P-chart and a C-chart. A P-chart could be used to monitor the percent of SKU's in a sample that the observed inventory level does not match the recorded inventory level. This means a random sample of n SKU's is selected and checked to see if the actual on hand inventory equals the recorded inventory. The numbers that don't equal each other is divided by the total sample size, which gives an estimate of the inventory accuracy, or P. Over time P could be plotted on a P chart. A C-chart could be used to monitor the collective number of item adjustments for a set of randomly observed SKU's where the on hand inventory failed to match the recorded inventory. For a C-chart, an inspection unit of size n is sampled, and the observed number of nonconformities is plotted in relationship to time.

This research focuses on SPC, specifically C-charts, as an acceptable substitute for cycle counting for monitoring inventory record accuracy. The research looks at a population, which is represented by a large retail store or equivalent environment with thousands of SKU's. The likelihood of a single SKU being accurate at a given time is assured to follow or follows a Bernoulli process, and when a given SKU if found to be inaccurate, the absolute value of the inventory adjustment follows a Poisson process. Each SKU within an operational unit can behave uniquely carrying its own Poisson rate.

This violates the basic assumption of SPC that each sampled unit follows the same distribution. Therefore we use a mixture distribution to comprehensively model each of these subpopulations. A C-chart monitors the number of inventory adjustments made to a specific SKU. From there, we inspect the Type I and Type II error rates of the C-chart.

2. Background Literature

This review provides an overview of literature involving cycle counting, SPC, and associated error rates.

2.1. Cycle Counting

[1] describes cycle counting as a process where trained individuals inspect a finite number of SKU's in inventory and compare the observed on hand items to the recorded inventory levels. These trained individuals examine the cause of the errors in the inventory and take corrective action when justified. Cycle counting is designed to comprehensively examine all the SKU's over time through sequential and collective counting exercises. The primary goals of cycle counting are to identify the causes of the errors, correct the conditions causing the errors, maintain a high level of inventory record accuracy, and provide a correct statement of assets [2]. [3] states that with the proper use of cycle counting, inventory record accuracy above 95% can be consistently maintained. [4] and [5] portray cycle counting as a quality assurance process that emphasizes correcting errors in inventory accuracy. There are various approaches of cycle counting including the geographical method [6], random sampling [3], the ABC method [7], [8], [9] and process control [2]. [1] says that you should not accept a one system fits all approach.   In complex operations, you may have a count program with multiple approaches, and the program should be customized to specific operational and business needs.

2.2. Statistical Process Control

It is well known that it is impossible to inspect or test the quality into a product; the product must be built right the first time. This implies that a process must be stable and that all individuals involved with the process must continually seek to improve process performance and reduce variability in key parameters [10]. [10] states that there is a four step process for Statistical Process Control; measure the process, eliminate undesirable variation from the process, monitor the process, and continuously improve the process. [10] also states that there are seven common tools that are used to monitor and improve processes: check sheet, pareto chart, flow chart, cause and effect diagram, histogram, scatter diagram, and control charts.

   The most commonly used of these tools is the control chart, and it is the tool that this paper is examining as a method to monitor inventory accuracy. The idea of a control

chart was first proposed by Walter Shewhart ([11] and [12].) A typical control chart plots the averages of measurements of a quality characteristic in samples taken from the process over time. The chart has a center line (CL) and upper and lower control limits (UCL and LCL). The center line represents the average of the process. If a given data point exceeds the thresholds as established by the UCL or LCL or other non-random trends appear, the process is declared out of control and operating with unknown sources of variability [10.]

2.3. Type I and Type II Errors for Control Charts

Control limits are generally set at 3 standard deviations away from the mean of the population. When a data point falls out of these limits, it indicates that the process is not stationary or out of control. There are two types of errors that are associated with control charts. They are type I and type II errors. The type I, alpha error known as the false alarm rate occurs when the null hypothesis is rejected, but it is actually true. This happens when a point is plotted outside of the control limits when the process is generating observations in accordance to the null hypothesis. This means the operator concludes that the process is out of control when it is in fact in control. Type II, Beta error known as the miss rate happens when we fail to reject the null hypothesis but the alternative is actually true. This means that the operator concluded the process was in control when it was in fact out of control.

The average run length (ARL) is the average number of points that is needed to detect an out of control condition. Large in-control ARLs and small out of control ARLs are desired. ARL calculations can become quite complicated as documented in the literature. For example, [13] models control charts using a Markov chain approach to evaluate the ARL when supplementary run rules are applied. [14] and [15] look at using the EWMA control chart instead of the X chart and evaluate different methods of computing the ARL properties of EWMA control charts. [16] studies the effect of using run rules on X charts. There are many other advances in the research of ARL as seen in the thousands of research advances documented in the literature.

3. Modeling Details

It is common in large industrial environments to have thousands SKUs in inventory or on retail shelves. Inventory adjustments required at the SKU level (when inventory is found to be inaccurate) is random from a process perspective and relatively rare. As such, we assume the number of inventory adjustments per SKU to follow a Poisson process. Comprehension of such complex composites of an entire population is effectively impossible to fathom as a typical retail environment could have thousands of SKUs. As the number of SKUs becomes large, measures for tractable modeling must be taken to collapse the individual SKU distributions to a manageable number of subpopulations. This collapsing process is justified by our observations of inventory adjustments in large

retail environments. For example, if you consider the electronic department in a large retail environment, individual DVDs, regardless of the title, behave statistically similar. Such observations have been made countless times in our investigation of large inventory systems.

As we will describe, we pursue several progressive venues to examine the nature of sampling from a population that is the result of many subpopulations. In essence, we make the assumption that our populations of inventory adjustments can be modeled as a set of subpopulations. As a result, the populations are mixture distributions of finite sets of subpopulations. In what follows, there is a high level discussion of the modeling efforts that include the approaches of enumeration and probability. Preliminary simulation results of scenarios using large samples sizes have been explored and are briefly introduced in the poster session.

3.1. Enumeration

We assume that an inventory system population can be represented by five sub-populations. Once the pdf is described as a mixture of five Poisson subpopulations, the expected value and variance for the populations are easily determined. With these values, the control limits of the classic C control chart can be calculated. Finally, the type I and II error rates for the example problems are obtained using complete enumeration. Solving these examples using complete enumeration creates a foundation of this work that supports extensions of the results by providing a confident basis for verification.

3.1.1. Method

The method used to solve the aforementioned examples of our problem is described in this section. This includes a numerical example that shows how the results are calculated. The notation used in our model to solve these problems is shown below:

$\propto_i$ = proportion of the population represented by pdf i
λ_i = Poisson arrival rate of the ith population
X_i = random variable representing the number of inventory adjustments from the sample population
Z_i = random variable representing subpopulation i
Y = random variable that consists of the sum of the values for X_i
n = number of SKU's sampled to assess inventory adjustments for the population
m = number of subpopulations (assumed five for this paper)

Aside from the notation, it is also important to note that the number of inventory adjustments, X_i, in a subpopulation is representative of the number of adjustments that have to be made for a specific SKU. For example, if inventory is checked and records show that there should be 55 units in stock and there are only 50 units, an inventory adjustment of 5 units would be required. When monitoring any process with sampled

observations, the collection of observations is often summarized by summing the individual observation for a given sampling period. For our effort, we call this random variable Y, and it follows this expression:

$$Y = X_1 + X_2 + .. + X_n \tag{1}$$

Where n represents the number of SKUs being sampled. X_i is assumed to be a mixture of m subpopulations and follows the pdf described in (2)

$$f_{Xi} = \propto_1 f_{z1} + \propto_2 f_{z2} + \cdots + \propto_m f_{zm} \text{ where, } \sum_{i=1}^{m} \alpha_i = 1 \tag{2}$$

Where Z_i is the variable of sub population i. This mixture population closely resembles mixture distributions found in popular probability texts [17].

3.1.2. Constructing an Example

The first step to solving our example problem is to determine the expected value and variance of a mixture distribution example. The expected value and variance are determined by using basic expected value operators and is shown in [18]. For our application, we use a mixture of m Poisson distributions, and the resulting expected value, $E(X_i)$ and variance, $\text{Var}(X_i)$ are as follows:

$$E(X_i) = \sum_{i=1}^{m} \propto_i E(Z_i) \tag{3}$$
$$Var(X_i) = \sum_{i=1}^{m} \propto_i [(\lambda_i)^2 + \lambda_i] - (\sum_{i=1}^{m} \propto_i \lambda_i)^2 \tag{4}$$

Where $\propto_i$ is the proportion of the population represented by the pdf i, and λ_i is the Poisson arrival rate of the ith population.

The next step is to determine what the control limits of this problem are. This is done using the following equations:

$$\text{UCL} = nE[X_i] + 3\sqrt{nVar[X_i]} \tag{5}$$
$$\text{LCL} = nE[X_i] - 3\sqrt{nVar[X_i]} \tag{6}$$

Where, n is the number of SKUs sampled, and X_i is a random variable that represents the number of inventory adjustments in the sample population.

Now that these equations have been introduced, an example with $m = 5$ subpopulations and $n = 2$ SKUs sampled is presented. The problem has the following parameters:

$\lambda_i = 0.25, 0.50, 0.75, 0.20, 0.30$
$\propto_i = 0.15, 0.20, 0.05, 0.40, 0.20$

Once these parameters are presented, $E(X_i)$ is computed according to (3), and it is determined to be 0.315. Similarly, the $\text{Var}(X_i)$ is computed according to (4), and it is 0.337. Next, an UCL = 4.74 and LCL = -2.22 are computed using (5) and (6), respectively.

Now that the control limits have been established, complete enumeration is used to exhaust all possible numerical combinations of the number of inventory adjustments for the two SKUs being sampled. For example, the only combination for $Y=0$ is $X_1=0$ and $X_2=0$. Therefore, the probability of $Y=0$ is simply the $\Pr(X_1=0)*\Pr(X_2=0)$. This process is carried out all the way to $Y=11$, which consists of 12 different combinations of X_1 and X_2. After enumerating $Y=11$, the cumulative probability sums to 1 using nine significant digits, and it can be concluded that practically all possible combinations of X_1 and X_2 have been exhausted. Table 1 presents the pdf for this scenario.

Table 1: Cumulative Probability for Example Problem

Cumulative Probability Example		
Y = X1 + X2	Prob(Y)	Cum Prob
0	0.54375953	0.54375953
1	0.32079783	0.864557361
2	0.104350788	0.968908149
3	0.025076815	0.993984964
4	0.004989009	0.998973973
5	0.000867797	0.99984177
6	0.000135831	0.999977601
7	1.94586E-05	0.999997059
8	2.58E-06	0.999999639
9	3.19167E-07	0.999999958
10	3.70631E-08	0.999999995
11	4.0588E-09	1

The final step is to determine the type I error rate for this example. The type I error rate calculation for this example is done using:

$$p' = 1 - (P[X_1 + X_2 \le UCL] - P[X_1 + X_2 < LCL]) \tag{7}$$

In this equation, if the LCL is calculated to be less than zero, it is set to zero. Our problem has an UCL = 4.74 and a LCL = 0. To find the type I error, all the possible combinations of $X_1 + X_2$ that are less than or equal to 4 must be summed and subtracted from 1. This is relatively easy considering that all of the possible combinations have already been enumerated. Using this information the type I error rate for our example turns out to be 0.001026.

3.2. Modeling Using Conditional Probability

This section examines using conditional probabilities to calculate both the type I and type II error rates of mixture distribution problems. The use of conditional probabilities greatly simplifies the calculations required to determine these errors, making its application to our problem advantageous. Applying conditional probabilities to solve type I and type II errors for mixture distributions serves as a practical method to computing these errors.

3.2.1. Modeling

This section describes the methods for which conditional probabilities are calculated. The derivation of equations for the type I error values for $n=2$, 3, 4, and 5 are shown as derived in [18] and [19]. First, when 2 random samples of SKUs are collected, the total number of inventory adjustments for the sample is simply the sum of X_1 and X_2 (or $n=2$, using our notation).

Using the UCL for a sample of 2,

$$P[X_1 + X_2 \leq UCL]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times P[X_2 \leq (UCL - X_1)] \tag{8}$$

Using the LCL for a sample of 2,

$$P[X_1 + X_2 < LCL]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] \times P[X_2 < (LCL - X_1)] \tag{9}$$

The probability of a type I error when $n=2$ is,

$$1 - p' = P[X_1 + X_2 \leq UCL] - P[X_1 + X_2 < LCL]$$

$$p' = 1 - (P[X_1 + X_2 \leq UCL] - P[X_1 + X_2 < LCL]) \tag{10}$$

Using the UCL for a sample of 3,

$$P[X_1 + X_2 + X_3 \leq UCL]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] P[X_2 + X_3 \leq (UCL - X_1)]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] P[X_3 \leq (UCL - X_1 - X_2)] \right] \tag{11}$$

Using the LCL for a sample of 3,

$$P[X_1 + X_2 + X_3 < LCL]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] P[X_2 + X_3 < (LCL - X_1)]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] \left[\sum_{X_2=0}^{LCL-X_1} P[X_2] P[X_3 < (LCL - X_1 - X_2)] \right] \tag{12}$$

Therefore, using the probability of being within the range of the control limits, the type I error rate, p', is,

$$1 - p' = P[X_1 + X_2 + X_3 \leq UCL] \text{ - } P[X_1 + X_2 + X_3 < LCL]$$

$$p' = 1 - (P[X_1 + X_2 + X_3 \leq UCL] \text{ - } P[X_1 + X_2 + X_3 < LCL]) \tag{13}$$

Using the UCL for a sample of 4,

$$P[X_1 + X_2 + X_3 + X_4 \leq UCL]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times P[X_2 + X_3 + X_4 \leq (UCL - X_1)]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] \times P[X_3 + X_4 \leq (UCL - X_1 - X_2)] \right]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] \times \left[\sum_{X_3=0}^{UCL-X_1-X_2} P[X_3] \times P[X_4 \leq (UCL - X_1 - X_2 - X_3)] \right] \right] \tag{14}$$

Using the LCL for a sample of 4,

$$P[X_1 + X_2 + X_3 + X_4 \leq LCL]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] \times P[X_2 + X_3 + X_4 \leq (LCL - X_1)]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] \times \left[\sum_{X_2=0}^{LCL-X_1} P[X_2] \times P[X_3 + X_4 \leq (LCL - X_1 - X_2)]\right]$$

$$= \sum_{X_1=0}^{LCL} P[X_1] \times \left[\sum_{X_2=0}^{LCL-X_1} P[X_2] \times \left[\sum_{X_3=0}^{LCL-X_1-X_2} P[X_3] \times P[X_4 \leq (LCL - X_1 - X_2 - X_3)]\right]\right] \quad (15)$$

Therefore, using the probability of being within the range of the control limits, the type I error rate, p', is,

$$1 - p' = P[X_1 + X_2 + X_3 + X_4 \leq UCL] - P[X_1 + X_2 + X_3 + X_4 < LCL]$$

$$p' = 1 - (P[X_1 + X_2 + X_3 + X_4 \leq UCL] - P[X_1 + X_2 + X_3 + X_4 < LCL]) \quad (16)$$

Using the UCL for a sample of 5,

$$P[X_1 + X_2 + X_3 + X_4 + X_5 \leq UCL]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times P[X_2 + X_3 + X_4 + X_5 \leq (UCL - X_1)]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] \times P[X_3 + X_4 + X_5 \leq (UCL - X_1 - X_2)]\right]$$

$$= \sum_{X_1=0}^{UCL} P[X_1] \times \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] \times \left[\sum_{X_3=0}^{UCL-X_1-X_2} P[X_3] \times P[X_4 + X_5 \leq (UCL - X_1 - X_2 - X_3)]\right]\right]$$

$$\sum_{X_1=0}^{UCL} P[X_1] \times \left[\sum_{X_2=0}^{UCL-X_1} P[X_2] \times \left[\sum_{X_3=0}^{UCL-X_1-X_2} P[X_3] \times \left[\sum_{X_4=0}^{UCL-X_1-X_2-X_3} P[X_4] \times P[X_5] \leq\right.\right.\right.$$

$$(UCL - X_1 - X_2 - X_3 - X_4)]]] \quad (17)$$

Using the LCL for a sample of 5,

10

$$P[X_1 + X_2 + X_3 + X_4 + X_5 \leq LCL]$$

$$= \textstyle\sum_{X_1=0}^{LCL} P[X_1] \times P[X_2 + X_3 + X_4 + X_5 \leq (LCL - X_1)]$$

$$= \textstyle\sum_{X_1=0}^{LCL} P[X_1] \times \left[\textstyle\sum_{X_2=0}^{LCL-X_1} P[X_2] \times P[X_3 + X_4 + X_5 \leq (LCL - X_1 - X_2)]\right]$$

$$= \textstyle\sum_{X_1=0}^{LCL} P[X_1] \times \left[\textstyle\sum_{X_2=0}^{LCL-X_1} P[X_2] \times \left[\textstyle\sum_{X_3=0}^{LCL-X_1-X_2} P[X_3] \times P[X_4 + X_5 \leq (LCL - X_1 - X_2 - X_3)]\right]\right]$$

$$= \textstyle\sum_{X_1=0}^{LCL} P[X_1] \times \left[\textstyle\sum_{X_2=0}^{LCL-X_1} P[X_2] \times \left[\textstyle\sum_{X_3=0}^{LCL-X_1-X_2} P[X_3] \times \left[\textstyle\sum_{X_4=0}^{LCL-X_1-X_2-X_3} P[X_4] \times P[X_5]\right. \right.\right. \leq$$

$$\left.\left.\left. (LCL - X_1 - X_2 - X_3 - X_4)\right]\right]\right] \tag{18}$$

Therefore, using the probability of being within the range of the control limits, the type I error rate, p', is,

$$1 - p' = P[X_1 + X_2 + X_3 + X_4 + X_5 \leq UCL] - P[X_1 + X_2 + X_3 + X_4 + X_5 < LCL]$$

$$p' = 1 - (P[X_1 + X_2 + X_3 + X_4 + X_5 \leq UCL] - P[X_1 + X_2 + X_3 + X_4 + X_5 < LCL]) \tag{19}$$

We have shown a probabilistic approach to determine the Type I and II errors for sample sizes of 2, 3, 4, and 5. To compute them, we developed a model in C++. The program computes the Type I errors for different values of $\propto$, λ, m, and n using conditional probabilities. To make changes to the values of $\propto$ and λ in the model, the user modifies input values. To alter the number of subpopulations, m, additional constants for $\propto$ and λ with input values for each must be created. Finally, to change the value of the number of SKUs sampled, n, the constant called *dim* in the code must be altered to represent the desired number of SKUs to be sampled. Using this code, the results for the Type I and II errors for problems containing $n=2$, 3, 4, and 5 are obtained.

4.   Results

4.1.   Problem Structures Considered

This section provides the type I errors for a balanced set of experimental conditions across values of λ_i and $\propto_i$ for a given m and n. These different structures are examined in the next section. The various problem structures presented are for examples consisting of $n=2$ and $n=3$ SKUs being sampled. These problem structures are designed to give a

broad representation of possible designs for real world mixture distributions when the number of subpopulations is five. The modifications made to each of these models consist of the following alterations to $\propto$ and λ:

$\propto$: Skewed Left (SL), Skewed Right (SR), and Fixed (F)

λ: Equal, Minimum–Maximum, Maximum–Minimum, Weighted Left (WL), Weighted Right (WR)

Examples of the individual parameter values of each of the scenarios can be seen below:

$\propto$ - SL: 0.025, 0.025, 0.025, 0.025, 0.9

SR: 0.9, 0.025, 0.025, 0.025, 0.025

F: 0.2, 0.2, 0.2, 0.2, 0.2

λ - Equal: 0.5, 0.5, 0.5, 0.5, 0.5

MinMax: 0.1, 0.2, 0.3, 0.4, 0.5

MaxMin: 0.5, 0.4, 0.3, 0.2, 0.1

WL: 0.9, 0.1, 0.1, 0.1, 0.1

WR: 0.1, 0.1, 0.1, 0.1, 0.9

We have taken these scenarios and developed a total of 15 different models to be examined for each value of n. The type I errors for each of these models are shown in the next section.

4.2. Results Using Enumeration

This section provides the results for all of the combinations of problem structures discussed in the previous section. Using these pdf's the type I error rates for $n=2$ can be summarized in Table 2.

Table 2: Type I error for $n=2$

Type I Error for n=2					
		Lambda			
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.003659847	0.016830027	0.029747787	0.029875375	0.008513164
	Skewed Right	0.003659847	0.029747787	0.016830027	0.008513164	0.029875375
	Fixed	0.003659847	0.004914922	0.004914922	0.010460924	0.010460924

This table shows that the combinations have type I errors that are very small, but they exceed the traditional view of .0027 under the classic assumption of normality. They compare favorably if the probability of a type I error is designed to be less than 5%. There are some differences in the type I errors, and it is important to note that all of the values are small enough to consider the model to be effective in terms of type I error for all of the different problem structures. There are also several patterns evident in this chart. First, is when the values of λ are equal, the type I error for all values of $\propto$ are the same. This logically makes sense because the E(X) and Var(X) will be the same for all three structures. Next, when the values of $\propto$ are fixed, the type I error for the Min-Max and Max-Min structures are the same because the calculations involved are the same, the numbers are just multiplied in reverse order. The same can be said for fixed $\propto$ values for the weighted left and weighted right structures. The final patterns in the chart are the diagonal values of the Min-Max and Max-Min, and Weighted Left and Weighted Right. These diagonal values are equal for both skewed left and skewed right $\propto$ values. Once again, this is because the numbers being multiplied are essentially the same as those being multiplied in reverse order. These patterns are useful to note, but the most important result of the graph is that all of these type I error values are significantly small, showing that the model is effective for a broad spectrum of problem structures. The type I error rates for *n*=3 are be summarized in Table 3.

Table 3: Type I error for *n*=3

Type I Error for n=3					
		Lambda			
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.004455981	0.003748813	0.008903452	0.01098578	0.005020188
	Skewed Right	0.004455981	0.008903452	0.003748813	0.005020188	0.01098578
	Fixed	0.004455981	0.016825077	0.016825077	0.024336803	0.024336803

This table shows that the type I errors for *n*=3 follow a very similar trend to the *n*=2 values. All of the patterns for the *n*=2 values are also evident in this chart. The most important conclusion that can be drawn is once again, that all of these type I errors are small enough to demonstrate that the model is effective for all of the different problem structures.

4.3. Results Using Conditional Probability

The first step in analyzing the results with this approach using the C++ program is to compare the results for *n*=2 and *n*=3 to the results that were obtained from complete enumeration for the Type I and II errors. As mentioned earlier, the complete enumeration results serve as a foundation to compare the results obtained from our program to confirm their accuracy. Results using the conditional probability approach are compared to the results using complete enumeration. Specifically, we compare the table of the type I errors obtained from the program to the table of type I errors obtained from the complete enumeration. The type I errors for all of the structures examined are nearly identical for both *n*=2 and *n*=3.

The next step is to increase the value of *n* to try and reach a more practical level. For this work, the value of *n* is increased to 4 and 5. The type I error rates for *n*=4 are summarized in Table 4.

Table 4: Conditional Probability Type I Errors for *n*=4

		Type I Error for n=4				
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
	Skewed Left	0.004328	0.003503	0.017428	0.019515	0.008041
Alpha	Skewed Right	0.004328	0.017428	0.003503	0.008041	0.019515
	Fixed	0.004328	0.010013	0.010013	0.01555	0.01555

This shows the type I errors for all the problem structures examined in this paper when the sample size is four. It is also evident that the patterns that were illustrated in the charts for *n*=2 and *n*=3 are still present. The type I error rates for *n*=5 are summarized in Table 5.

Table 5: Conditional Probability Type I Errors for *n*=5

		Type I Error for n=5				
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
	Skewed Left	0.003989	0.003126	0.005423	0.007335	0.00611
Alpha	Skewed Right	0.003989	0.005423	0.003126	0.00611	0.007335
	Fixed	0.003989	0.005951	0.005951	0.009955	0.009955

The results for *n*=4 and *n*=5 show that our method provides small values for the Type I error rates. Furthermore, these values illustrate a tendency to decrease as the value of *n*

increases. This is significant, because in practice, the method of using a mixture distribution to monitor inventory accuracy would likely involve using higher values of *n*. Low type I error are realized, and as expected, the rate continues to decrease as the samples size increases.

We now investigate the type II error rates of small and large shifts for *n* = 2, 3, 4, and 5. This is done by assuming that the underlying distribution is going to be the model of fixed ∝ values and equal λ values (assumed "in-control" condition), and the type II errors are determined by overlaying each scenario on the assumed "in-control" condition. Making this assumption provides a basis to compute the type II error rates. This does not provide a complete examination of all of the type II error rates, but it presents an idea of how these models perform in terms of type II errors.

The type II errors for *n*=2 are summarized in Table 6.

Table 6: Type II Errors for *n*=2

		Type II Error for n=2				
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
	Skewed Left	0.996340153	0.996810259	0.999958387	0.999796682	0.96985181
Alpha	Skewed Right	0.996340153	0.999958387	0.996810259	0.96985181	0.999796682
	Fixed	0.996340153	0.999255895	0.999255895	0.997371137	0.997371137

As seen in this table, the type II errors are all relatively large and demonstrate that small shifts in λ are hard to detect. The type II errors for *n*=3, 4, and 5 respectively are summarized in the following tables:

Table 7: Type II Errors for *n*=3

		Type II Error for n=3				
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
	Skewed Left	0.995544019	0.996251187	0.999982229	0.999905216	0.955404222
Alpha	Skewed Right	0.995544019	0.999982229	0.996251187	0.955404222	0.999905216
	Fixed	0.995544019	0.999365281	0.999365281	0.997905047	0.997905047

Table 8: Type II Errors for *n*=4

Type II Error for n=4					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.995672	0.996497	0.999999	0.999999	0.944491
	Skewed Right	0.995672	0.999999	0.996497	0.944491	0.999999
	Fixed	0.995672	0.999657	0.999657	0.999999	0.999999

Table 9: Type II Errors for *n*=5

Type II Error for n=5					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.996011	0.996874	0.999999	0.999999	0.936976
	Skewed Right	0.996011	0.999999	0.996874	0.936976	0.999999
	Fixed	0.996011	0.999831	0.999831	0.999999	0.999999

These tables show that the performance is similar to the results for *n*=2. This gives us an idea of how our model performs in terms of type II errors.

To represent a large shift, the values for λ in each of our problem structures will be multiplied by 3. This means that if the MinMax problem structure is multiplied by 3, the shifted values would change to .3, .6, .9, 1.2, 1.5. For these large shifts, we once again assume that the underlying distribution is going to be the model of fixed $\propto$ values and equal λ values (at the original values,) but this time the actual distributions are assumed to be the inflated values of 3 times the original λ values for each structure. The following tables present the type II errors for *n*=2, 3, 4, and 5 and inflated values of 3 times the original λ values.

Table 10: λ Inflated 3 Times Type II Error for *n*=2

λ Inflated 3 Times Type II Error for n=2					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
	Skewed Left	0.815263245	0.835176042	0.996033985	0.990227394	0.459125164
Alpha	Skewed Right	0.815263245	0.996033985	0.835176042	0.459125164	0.990227394
	Fixed	0.815263245	0.948812464	0.948812464	0.915564198	0.915564198

Table 11: λ Inflated 3 Times Type II Error for *n*=3

λ Inflated 3 Times Type II Error for n=3					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.702930435	0.737237134	0.996249783	0.990395103	0.277976337
	Skewed Right	0.702930435	0.996249783	0.737237134	0.277976337	0.990395103
	Fixed	0.702930435	0.926219341	0.926219341	0.899825346	0.899825346

Table 12: *λ* Inflated 3 Times Type II Error for *n*=4

λ Inflated 3 Times Type II Error for n=4					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.606680	0.652683	0.996957	0.991967	0.168131
	Skewed Right	0.606680	0.996957	0.652683	0.168131	0.991967
	Fixed	0.606680	0.908847	0.908847	0.889443	0.889443

Table 13: *λ* Inflated 3 Times Type II Error for *n*=5

λ Inflated 3 Times Type II Error for n=5					
		Lambda				
		Equal	Min-Max	Max-Min	Weighted Left	Weighted Right
Alpha	Skewed Left	0.525048	0.582937	0.997601	0.993082	0.101876
	Skewed Right	0.525048	0.997601	0.582937	0.101876	0.993082
	Fixed	0.525048	0.894962	0.894962	0.881996	0.881996

These tables confirm that our model is much more effective at detecting type II errors when the shift is larger. In fact, there is more than a 50% chance that shifts of these orders will be detected with a sample of five on the very next sample. These tables also show the trend of the type II errors decreasing as the sample size *n* increases.

5.   Conclusions

There are several conclusions that are drawn after completely enumerating the mixture distribution problem. The first is that solving example problems with complete enumeration establishes a foundation for further analysis. Specifically, the sample sizes of 4 or 5 become quite detailed; therefore, we switch from complete enumeration to using probability as shown in [18] and [19]. The results from complete enumeration were utilized to validate the [18] models as deployed in a C++ program. In our results, we examine a wide variety of mixture distribution structures. Inspecting these different

structures shows that there are several patterns or similarities that can be identified between different structures. In essence, we find that our approach to using the C chart as a means for monitoring inventory adjustments yields very low Type I errors and provides a great deal of sensitivity to detection of significant changes. Our analysis as presented here is limited to samples sizes of five, and we realize that extensions to sample sizes of 25 to 100 will be more reasonable in application.

At the poster session, we will present some preliminary findings based upon simulation results that demonstrate the effectiveness of the SPC approach to more realistic situations. We continue to work closely with the industry to investigate and support the application of this approach.

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