A NEW MILP APPROACH FOR THE FACILITY LAYOUT DESIGN PROBLEM WITH RECTANGULAR AND L/T SHAPED DEPARTMENTS

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Abstract

In this paper we propose a new approach for the facility layout problem (FLP) and suggest new mixed-integer linear programming (MILP) formulations. The proposed approach considers simultaneously the location of the departments within the facility and the internal arrangement of the machines. Two models are suggested, where the first addresses the rectangular department case and the second allows non-rectangular departments defined by an L/T shape. New regularity constraints are developed to avoid irregular department shapes.

1   Introduction and Literature Review

The facility layout problem (FLP) addresses the allocation of space area to components of a facility, such as, machines, material handling equipment, aisles, storage areas, etc. The objective function is typically related to transportation costs or non-quantitative closeness performance measures between depatrmets. Traditionally, most approaches for the FLP have followed the systematic layout planning (SLP), which was introduced in [1]. As typical engineering design problems, SLP is based on a hierarchical approach, in which the area is first assigned to departments, and then the same approach is repeated for each department separately, to assign area to each of its components. This approach is also called a top-down approach [2], as the block layout problem is solved in the first stage and the detailed layout design is addressed in the second stage.

Most of the research presented in the literature deals with the first stage of the hierarchical approach. Various methods have been suggested to divide a given area among different departments, assuming that the detailed design will be done later on. These methods may be divided into two types. The first assumes a discrete area, where the total area is divided into relatively small squares. Then, various heuristics may be applied to allocate each square to a specific department (see [3] and [4]). The second assumes a continuous area, where the total area is divided among the departments using meta heuristics or mixed-integer programming (MIP) formulations. Our solution approach assumes similarly a continuous area, but solves simultaneously the detailed

design problem of allocating the machines, so that the full plan is obtained. We propose a new MILP formulation to solve the resulting problem.

The first MIP formulation for the facility layout problem was presented by Montreuil [5]. This model addresses the problem of positioning a set of departments in a rectangular facility, where the area size of each department is given, its final shape is rectangular and the dimensions are decision variables. A periodic flow of material between each pair of departments is given, and the objective is to minimize the total transportation costs within the facility, assuming linearity of the costs with the amount of flow and the distance traveled.

Three difficulties are associated with the above formulation, one is technical and the other two are conceptual. The technical problem refers to the non-linearity of the departments' area-related constraints. Some progress has been made with respect to this issue by Sherali et al. [6] by providing piecewise linear approximation methods.

The second difficulty is related to the nature of the hierarchical approach mentioned above. This approach ignores the characteristics of the components to be located later on in each department, after its area has been fixed. In particular, it does not take into consideration the number and dimensions of the machines placed in each department. This issue is discussed in [2] who suggest a formulation which takes into account several configurations of each department as well as the material handling cost between output and input points of the departments. Similar to previous research, only rectangular shape departments are considered. The formulation also involves the sequence-pair concept for improving its tractability [7]. The third difficulty is related to the rectangular shape constraint imposed on the departments. This restriction is often unnecessary and excludes high quality solutions from consideration.

In this paper we suggest a new approach for the facility layout design problem, where the number and dimensions of machines in each department, rather than the department area, are given as an input data. A pre-processing stage is performed to generate all non-dominated configurations of the departments, based on varying internal arrangements of the machines. Then, two mixed-integer linear programming (MILP) formulations are developed to find the final layout, based on the data generated in the pre-processing stage. The first formulation proposed in this paper addresses the rectangular department case. Next, this formulation is extended to the non-rectangular case. In the latter, a general L/T department shape is suggested, where each department consists of two rectangles, containing together the given number of machines. To avoid irregular department shapes, these rectangles are forced to be connected in a way that provides a shape similar to the letter L or T. Another measure to avoid irregularity is developed for the L/T shaped departments, which can be viewed as an extension of the aspect ratio, commonly used for rectangular shapes. In particular, the length in each dimension of the smallest enclosing rectangle is bounded as well as the sum of both dimensions.

The rest of the paper is organized as follows. In Section 2 we provide a detailed description of the problem and present pre-processing steps and preliminary results, which are useful to formulating and solving the problem. In Section 3 we provide a

formulation of the first variant of the problem, in which the shape of all departments is restricted to be a rectangle, hence referred to as the rectangle-shape problem. In Section 4 we extend the formulation and analysis to the non-rectangular, L/T shaped departments. Finally, Section 5 concludes the paper.

2   Problem Description and Preliminaries

A rectangular facility of given length and width dimensions need to be designed so that a set of departments is located in it. Machines need to be placed within the area associated with each department, subject to certain restrictions on the area shape. Within each department all machines are identical; however the number of machines and their dimensions are department-dependent. Finally, the flow of material between each pair of departments is given.

The decisions that need to be made in this problem consist of where to position each department within the facility, subject to the above mentioned constraints. Note that the area size that each department occupies is not pre-determined, but depends on the internal positioning of the machines within it, which needs to be determined as well. The objective is to minimize the total transportation costs in the facility, assuming it is obtained by summing up the products of the flow between each pair of departments and the rectilinear distance between the centroids of these departments.

As discussed in the Introduction, we consider either rectangle-shape departments or L/T shape departments. The latter is the case where each department consists of two rectangles, connected in a way that forms an L or a T shape, as explained in more details in Section 4 below. Thus, in both cases, possible rectangle shape departments need to be generated. This is achieved through a pre-processing step as described below.

We first present the problem's input parameters, based on which we perform the pre-processing step, and generate additional parameters which are used in the problem formulation.

Parameters of the problem

I number of departments
m_i number of machines in department i , $i = 1,...,I$
a_i the dimension of a machine in department i along the x-axis in the original
        orientation
b_i the dimension of a machine in department i along the y-axis in the original
        orientation
A the dimension of the facility along the x-axis
B the dimension of the facility along the y-axis
f_{ij} flow between departments i and j

As mentioned above, we consider solutions to the layout problem, where department shapes are based on rectangles. In the basic case, the entire department has a rectangular shape, thus we first compute, for each department i, dimensions of possible rectangles which consist of m_i machines. We refer to each such possibility as a configuration. Later we show how this method is extended to generate L/T configurations, which are made of two rectangles.

Rectangles consisting of m_i machines

We assume that within each rectangle, machines may be placed either in their original orientation, or in a 90^0 rotated orientation. No other orientation is allowed, and all machines within a certain rectangle have to be placed in the same orientation. This leads to the following $2m_i$ possible configurations of rectangles, which consist each of m_i machines for department i.

Configuration r for $r = 1 \dots m_i$ is defined as a configuration, when all machines are positioned in their original orientation, and the row with the largest number of machines contains r machines. Configuration r for $r = m_i + 1 \dots 2m_i$ is defined as a configuration, when the machines are in their rotated orientation, and the row with the largest number of machines contains $r - m_i$ machines. Note that the dimensions of a rectangle are determined by the length of its longest row in the x-dimension and its longest row in the y-dimension. Thus, the dimensions of the above configurations are computed as follows:

A_{ir} is the size along the x axis of a rectangle of department i, when choosing configuration r, $r = 1 \dots 2m_i$, where

$$A_{ir} = \begin{cases} a_i \cdot r & r = 1, \dots, m_i \\ b_i \cdot (r - m_i) & r = m_i + 1, \dots, 2m_i \end{cases}.$$

B_{ir} is the size along the y axis of a rectangle of department i, when choosing configuration r, $r = 1, \dots, 2m_i$, where

$$B_{ir} = \begin{cases} \left\lceil \frac{m_i}{r} \right\rceil \cdot b_i & r = 1, \dots, m_i \\ \left\lceil \frac{m_i}{r - m_i} \right\rceil \cdot a_i & r = m_i + 1, \dots, 2m_i \end{cases}.$$

Dominated configurations

The above specified configurations include all $2m_i$ possibilities, arising from two possible orientations of the machines for each of m_i possibilities of the largest number of machines in the width (x axis) dimension. Note that the number of machines in one dimension determines the number of machines in the other dimension. However, some of these configurations are dominated by others with respect to the dimensions of the

departments, and can be removed from consideration. Specifically, we have the following dominance definition for two rectangles that have the same number of machines.

Definition 1: Configuration r_g which consists of m_i machines for department i is dominated by configuration r_h which consists of m_i machines for department i if: $A_{ir_g} \geq A_{ir_h}$ and $B_{ir_g} \geq B_{ir_h}$.

Example 1: Let $a_i=2$, $b_i=1$, $m_i=4$. Then, the following four configurations are created with respect to the original orientation:

r	A_{ir}	B_{ir}
1	2	4
2	4	2
3	6	2
4	8	1

Let $r_g=3$ and $r_h=2$. Then, $6 = A_{i3} > A_{i2} = 4$ and $2 = B_{i3} = B_{i2} = 2$, thus configuration 3 is dominated by configuration 2.

Note that dominance can also occur between an original and a rotated configuration. In the rotated configurations of the above example, all but the first configuration (1X8) are dominated by configurations of the original orientation.

Irregular configurations

We assume that department shapes need to satisfy some regularity conditions. Hence, configurations which violate these conditions may be removed. However, these considerations are made separately for the rectangle and the L/T shape departments, and thus are described in the respective sections.

3 Rectangle shape departments - model and formulation

In this section we require that each department will have a rectangular shape. For rectangular shape departments, a commonly used regularity condition in the literature is the aspect ratio (AR), a parameter which bounds the ratio of the two rectangle dimensions (width and length). Thus, adding the AR parameter to the input data of the rectangle shape problem, we use it in the pre-processing step to remove from considerations rectangle shapes, created as explained above, which violate this bound. This step is performed on the remaining configurations after the removal of the dominated ones.

Let R_i be the set of non-dominated rectangle configurations consisting of m_i machines, which satisfy the AR conditions.

For each department i, one configuration from the set R_i has to be selected. Under these assumptions, we define appropriate decision variables and present a MILP formulation, referred to as MILP-R.

Decision Variables for MILP-R

$w_{ir} = 1$, if department i is arranged in configuration r, ($w_{ir} = 0$, otherwise), $i = 1, ..., I$, $r = 1, ..., 2m_i$;

$z_{ij}^x = 1$, if department i precedes department j along the x direction, ($z_{ij}^x = 0$, otherwise), $i = 1, ..., I, j = 1, ..., I, j \neq i$;

$z_{ij}^y = 1$, if department i precedes department j along the y direction, ($z_{ij}^y = 0$, otherwise), $i = 1, ..., I, j = 1, ..., I, j \neq i$;

x_i the centroid of department i in the x direction, $i = 1, ..., I$;

y_i the centroid of department i in the y direction, $i = 1, ..., I$;

x_{ij} the distance along the x-axis between the centoids of departments i and j, $i = 1, ..., I, j = 1, ..., I, j \neq i$;

y_{ij} the distance along the y-axis between the centoids of departments i and j, $i = 1, ..., I, j = 1, ..., I, j \neq i$;

x_i^l the lower end of department i along the x-axis, $i = 1, ..., I$;

x_i^h the higher end of department i along the x-axis, $i = 1, ..., I$;

y_i^l the lower end of department i along the y-axis, $i = 1, ..., I$;

y_i^h the higher end of department i along the y-axis, $i = 1, ..., I$;

MILP-R:

$$\min \sum_{i \in I} \sum_{j \in I, j \neq i} f_{ij}(x_{ij} + y_{ij}) \tag{1}$$

s.t.

$$x_{ij} \geq x_i - x_j \qquad \forall i, \forall j, j \neq i \tag{2}$$

$$x_{ij} \geq x_j - x_i \qquad \forall i, \forall j, j \neq i \tag{3}$$

$$y_{ij} \geq y_i - y_j \qquad \forall i, \forall j, j \neq i \tag{4}$$

$$y_{ij} \geq y_j - y_i \qquad \forall i, \forall j, j \neq i \tag{5}$$

$$x_i = \frac{x_i^h + x_i^l}{2} \quad \forall i \qquad \forall i \tag{6}$$

$$y_i = \frac{y_i^h + y_i^l}{2} \quad \forall i \qquad \forall i \tag{7}$$

$$x_i^h - x_i^l \geq \sum_{r \in R_i} A_{ir} w_{ir} \qquad \forall i \tag{8}$$

$$y_i^h - y_i^l \geq \sum_{r \in R_i} B_{ir} w_{ir} \qquad \forall i \tag{9}$$

$$\sum_{r \in R_i} w_{ir} = 1 \qquad \forall i \tag{10}$$

$$x_i^h \leq x_j^l + A(1 - z_{ij}^x) \qquad \forall i, \forall j, i \neq j \tag{11}$$

$$y_i^h \leq y_j^l + B(1 - z_{ij}^y) \qquad \forall i, \forall j, i \neq j \tag{12}$$

$$z_{ij}^x + z_{ji}^x + z_{ij}^y + z_{ji}^y \geq 1 \qquad \forall i, \forall j, i \neq j \tag{13}$$

$$x_i^h \leq A \qquad\qquad \forall i \qquad\qquad\qquad (14)$$
$$y_i^h \leq B \qquad\qquad \forall i \qquad\qquad\qquad (15)$$
$$w_{ir} \in \{0,1\} \qquad\qquad \forall i, \forall r \in R_i \qquad\qquad (16)$$
$$z_{ij}^x, \ z_{ij}^y \in \{0,1\} \qquad\qquad \forall i, \forall j, i \neq j \qquad\quad (17)$$
$$x_i, y_i, x_i^l, x_i^h, y_i^l, y_i^h \ \geq 0 \qquad \forall i \qquad\qquad\qquad (18)$$
$$x_{ij}, y_{ij} \geq 0 \qquad\qquad \forall i, \forall j, i \neq j \qquad\quad (19)$$

The objective term (1) to be minimized includes the total transportation costs in the facility, obtained by multiplying the total flow with the rectilinear distance between each department pair and taking the sum over all pairs. Constraints (2)-(5) define the distances along the x- and y- axes between pairs of departments as the distance between the departments' centroids. Constraints (6)-(7) define the centorid of each department as the middle of the low and high ends of the department along both axes. Constraints (8) and (9) define the minimal distance between the high and low ends of each department to be no less than the width and length, respectively, of the chosen configuration for that department. Constraint (10) states that exactly one configuration from the set of configurations R_i should be chosen for each department. Constraints (11)-(12) ensure that the precedence relationship between departments i and j along the x and y directions, respectively, is satisfied with respect to department i's high end and department j's low end positions. Constraint (13) ensures that departments i and j do not overlap, by requiring that at least one precedence relationship exists between them. Constraints (14)-(15) require that departments are positioned within the facility dimensions. Finally, constraints (16)-(17) and constraints (18)-(19) define the problem variables to be binary and continuous, respectively.

4   L/T - shape - Model and Formulation

In this section we relax the requirement to construct rectangle-shape departments and allow them to be of an L/T shape. The department's L/T shape is created by connecting two rectangles in a way that forms an L or a T shape. In particular, we have the following definition.

Definition 2: an L/T shape is a shape created by two rectangles such that one edge of one of the rectangles is attached along its entire length to one of the edges of the other rectangle.
Figure 1 presents a layout which consists of several L/T departments. One can see that department 2 is rectangular, department 1 is of T shape and departments 3, 4 and 5 are of L shape. The rounded rectangles within each department denote the machines, and the different shades distinguish between the internal two rectangles of each department.
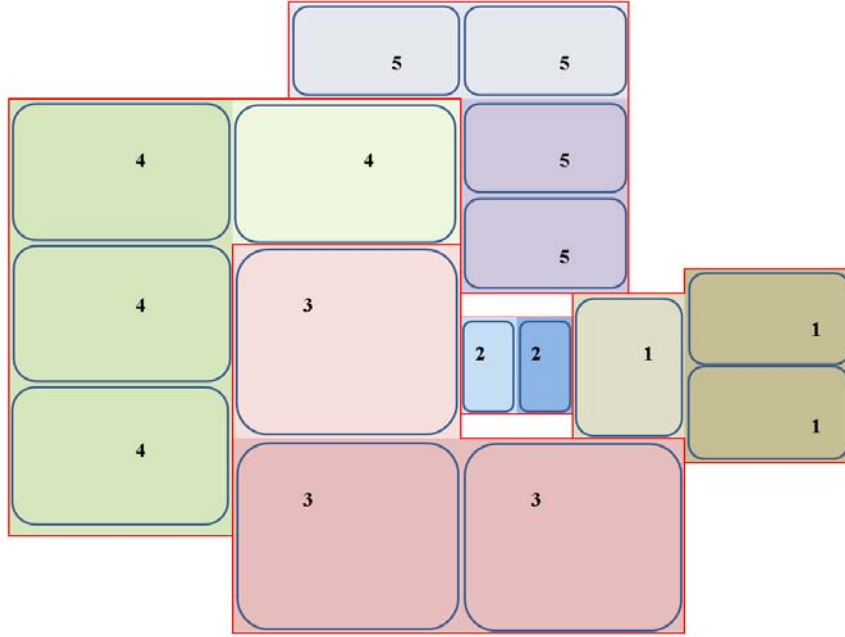
Figure 1: An example of an L/T layout solution

We create L/T shape departments by defining, for each department i, rectangle shapes that consist of up to m_i machines. Then two rectangles, instead of one, are selected for each department, such that the total number of machines in both of them is m_i. We further make sure through the MILP formulation that they are connected as described in Definition 2. To achieve this goal, we extend some of the definitions and procedures described in the previous section, as detailed next.

In the pre-processing step we generate, for each department i, all rectangle shapes consisting of up to m_i machines. When a rectangle consists of, say, l machines, all configurations with l machines are generated in exactly the same way as explained in the previous section for m_i machines. Thus, $2l$ configurations are generated for all $l = 1, \ldots, m_i - 1$, each consisting of l machines, for a total of $2 \sum_{l=1}^{m_i-1} l = m_i(m_i - 1)$ configurations. This number is later reduced by removing dominated configurations. L/T shapes with m_i machines are then obtained by selecting two configurations, each with $1 \le l \le m_i - 1$ machines, such that their sum equals m_i. In this section we use the term configuration (and the index r) to denote a rectangle with $1 \le l \le m_i - 1$ machines, rather than the final department's shape.

The configuration dimensions A_{ir} and B_{ir} have now the following meaning:
A_{ir} is the size of configuration r of department i along the x axis,
B_{ir} is the size of configuration r of department i along the y axis,
and we use l_r to denote the number of machines contained in configuration r.

The dominance definition is extended to the case where two configurations may include a different number of machines.

Definition 3: Configuration r_g which consists of l_{r_g} machines for department i is dominated by configuration r_h which consists of l_{r_h} machines for department i if: $l_{r_g} \leq l_{r_h}$, $A_{ir_g} \geq A_{ir_h}$ and $B_{ir_g} \geq B_{ir_h}$.

Example 2: Let $a_i=2$, $b_i=1$, $m_i=6$. Then, a configuration r_g with four machines ($l_{r_g} = 4$) may have $A_{ir_g} = 6$ and $B_{ir_g} = 2$ (see the left configuration in Figure 2), where a configuration r_h with five machines ($l_{r_h} = 5$) may have identical dimensions: $A_{ir_h} = 6$ and $B_{ir_h} = 2$. (see the right configuration in Figure 2). Consequently, configuration r_g is dominated by configuration r_h.
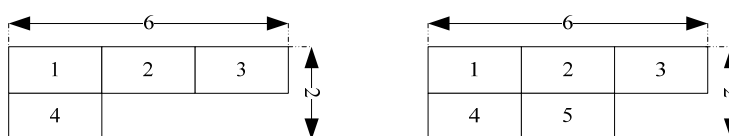

Figure 2: Example of Definition 3

It can be shown that removing dominated rectangles according to the dominance relation described in Definition 3, does not preclude any non-dominated L/T shapes from consideration.

Irregular configurations

No regularity condition specific for L/T-shapes is known in the literature, although several regularity conditions are defined with respect to general shapes, see [8] and [9]. Here we define new regularity conditions that are designed to handle specifically the L/T shapes considered in this work.

We first note that contrary to the rectangle-shape problem, here we do not remove in the pre-processing stage rectangle configurations that do not satisfy the aspect ratio condition. This is due to the fact that after connecting two rectangles, the final shape may meet regularity conditions, while the separate rectangles may not. Instead, we formulate two regularity conditions with respect to the entire department's width and length, rather than to the separate rectangles. For that purpose we define the length of department i with respect to its x and y axis, LA_i and LB_i, respectively, as the dimensions of the smallest enclosing rectangle (SER) of the department. This is illustrated in Figure 3 for three departments.
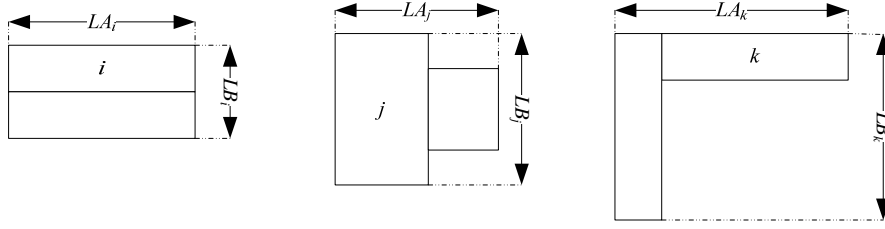
Figure 3: Examples of the department dimensions

The first regularity condition is motivated by the standard aspect ratio (AR) condition. When the length of a department along, say, its y-axis, is obtained by dividing the total machine area in the length along its x-axis (referring to the best case in terms of regularity), we require that the ratio between the latter (the numerator) and the former (the denominator) should be bounded by the AR parameter. Thus,

$$\frac{LA_i}{\frac{m_i a_i b_i}{LA_i}} \leq AR \implies LA_i \leq \sqrt{AR \cdot m_i a_i b_i}$$

where $ma_i b_i$ is the total machine area. This (and a similar restriction on LB_i) imposes an upper bound on the length of each dimension of the department.

The second condition is required since with non-rectangular shapes, irregular shapes can be obtained even when satisfying the aspect ratio constraint (see for example department k in Figure 3). To handle that, we add an additional constraint, which refers to the ratio of the area of the smallest enclosing rectangle (*SER*) of the department and the actual area of the department. This ratio is required to be bounded by a parameter, which we denote here by *PF* (as it will result in a perimeter constraint, or perimeter factor, see below). Note that the parameter *PF* should be strictly smaller than *AR*, since otherwise the second condition is always satisfied when the first condition holds for both dimensions. Thus, we require:

$$\frac{LA_i LB_i}{m_i a_i b_i} \leq PF$$

and noting that the numerator on the left side will be maximal, for a given perimeter length, when $LA_i = LB_i$, we approximate this non-linear constraint by applying the resulting condition to the sum of both dimensions, and obtain:

$$LA_i + LB_i \leq 2\sqrt{PF \cdot m_i a_i b_i}.$$

Requiring the above two conditions ensures that shapes of departments will not be unusually irregular, and this is achieved through the MILP formulation, where these requirements are added as constraints.

A final note is with respect to the objective function of the MILP formulation. Since each department is combined now of two rectangles formed in an L/T shape, it is less trivial to find the centroid point of it. We overcome this difficulty by keeping track of the

number of machines included in each configuration, and calculating the centroid point by using this information. To do this, we define for all $i = 1, \ldots, I, k = 1, \ldots, m_i - 1$:

R_{ik} = set of non-dominated rectangle configurations for department i which contain k machines.

We observe that when choosing for some department i two configurations, together consisting of m_i machines, they will be chosen from different R_{ik} sets, unless m_i is an even number, in which case two configurations may be selected from the set $R_{im_i/2}$. The latter case imposes some technical difficulties, which we are able to handle through variable duplication; however we ignore here this difficulty and assume, w.l.o.g. that m_i is an odd number.

We are now ready to present our MILP formulation for the L/T shape department case, referred to as MILP-LT.

Model formulation for L/T-shape departments

As discussed above, the MILP-LT formulation uses, in addition to the parameters used in the MILP-R formulation, the following parameters:

AR aspect ratio
PF perimeter factor
M   large number

The decision variables are extended relative to those used in the MILP-R formulation:

x_i, y_i, x_{ij}, y_{ij} are identical to those used in MILP-R;

The variables w_{ir} have the same meaning as previously, with respect to a configuration which is part of a department:

$w_{ir} = 1$, if configuration r of department i is chosen (=0, otherwise), $i = 1, \ldots, I, r \in \cup_k R_{ik}$.

Analogous to the variables $x_i^h, x_i^l, y_i^h, y_i^l$ in the rectangle case, each variable is now duplicated, to indicate the number of machines included in the configuration:

x_{ik}^h, x_{ik}^l the higher and lower end of configuration i along the x-axis, when the configuration contains k machines, $i = 1, \ldots, I, k = 1, \ldots, m_i - 1$, respectively;

y_{ik}^h, y_{ik}^l, are defined similarly, along the y-axis;

The analogous to the variables z_{ij}^x, z_{ij}^y are:

$z_{ik_1jk_2}^x = 1$, if a configuration of department i which includes k_1 machines precedes a configuration of department j which includes k_2 machines in the x direction, $i \neq j, \forall k_1 = 1, \ldots, m_i - 1, k_2 = 1, \ldots, m_j - 1$;

$z_{ik_1ik_2}^x = 1$, if a configuration of department i which includes k_1 machines precedes a configuration of the same department i which includes k_2 machines in the x direction, $\forall i, \forall k_1, k_2 = 1, \ldots, m_i - 1$;

$z_{ik_1jk_2}^y$ and $z_{ik_1ik_2}^y$ are defined similarly, along the y-axis;

$\bar{x}_{ik}$ = the centroid of a chosen configuration of department i in the x direction, when this configuration contains k machines, $i = 1, ..., I$, $k = 1, ..., m_i - 1$;

$\bar{y}_{ik}$ = the centroid of a chosen configuration of department i in the y direction, when this configuration contains k machines, $i = 1, ..., I$, $k = 1, ..., m_i - 1$;

$\tilde{A}_i$ = the length of the configuration which is the longest along the x-axis of the two chosen configurations of department i;

$\tilde{B}_i$ = the length of the configuration which is the longest along the y-axis of the two chosen configurations of department i;

s_i = 1 when the length of the department is equal to the length of the largest chosen configuration along the x direction (= 0 when the length of the department is equal to the length of the largest chosen configuration along the y direction);

LA_i = the length of department i along the x-axis;

LB_i = the length of department i along the y-axis;

$$\min \sum_{i \in I} \sum_{j \in I, j \neq i} f_{ij}(x_{ij} + y_{ij}) \tag{20}$$

$$x_{ij} \geq x_i - x_j \qquad \forall i, \forall j, j \neq i \tag{21}$$

$$x_{ij} \geq x_j - x_i \qquad \forall i, \forall j, j \neq i \tag{22}$$

$$y_{ij} \geq y_i - y_j \qquad \forall i, \forall j, j \neq i \tag{23}$$

$$y_{ij} \geq y_j - y_i \qquad \forall i, \forall j, j \neq i \tag{24}$$

$$x_i = \sum_{k=1}^{m_i-1} \frac{k\bar{x}_{ik}}{m_i} \qquad \forall i \tag{25}$$

$$y_i = \sum_{k=1}^{m_i-1} \frac{k\bar{y}_{ik}}{m_i} \qquad \forall i \tag{26}$$

$$\bar{x}_{ik} = \frac{x_{ik}^h + x_{ik}^l}{2} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{27}$$

$$\bar{y}_{ik} = \frac{y_{ik}^h + y_{ik}^l}{2} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{28}$$

$$\bar{x}_{ik} \leq M \sum_{r \in R_{ik}} w_{ir} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{29}$$

$$\bar{y}_{ik} \leq M \sum_{r \in R_{ik}} w_{ir} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{30}$$

$$x_{ik}^h - x_{ik}^l \geq \sum_{r \in R_{ik}} A_{ir} w_{ir} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{31}$$

$$y_{ik}^h - y_{ik}^l \geq \sum_{r \in R_{ik}} B_{ir} w_{ir} \qquad \forall i, \forall k = 1, ..., m_i - 1 \tag{32}$$

$$\sum_{k=1}^{m_i-1} \sum_{r \in R_{ik}} w_{ir} = 2 \qquad \forall i \tag{33}$$

$$\sum_{k=1}^{m_i-1} \sum_{r \in R_{ik}} k w_{ir} = m_i \qquad \forall i \tag{34}$$

$$x_{ik_1}^h \leq x_{jk_2}^l + A\left(1 - z_{ik_1 jk_2}^x\right) \qquad \forall i, \forall j \neq i, \forall k_1 = 1, ..., m_i - 1, k_2 = 1, ..., m_j - 1 \tag{35}$$

$$x_{ik_1}^h \leq x_{ik_2}^l + A\left(1 - z_{ik_1 ik_2}^x\right) \qquad \forall i, k_1 = 1, ..., m_i - 1, k_2 = 1, ..., m_i - 1, k_1 \neq k_2 \tag{36}$$

$$y_{ik_1}^h \leq y_{jk_2}^l + B\left(1 - z_{ik_1 jk_2}^y\right) \qquad \forall i, \forall j \neq i, \forall k_1 = 1, ..., m_i - 1, k_2 = 1, ..., m_j - 1 \tag{37}$$

$$y_{ik_1}^h \leq y_{ik_2}^l + B\left(1 - z_{ik_1 ik_2}^y\right) \qquad \forall i, k_1 = 1, ..., m_i - 1, k_2 = 1, ..., m_i - 1, k_1 \neq k_2 \tag{38}$$

$$z_{ik_1 jk_2}^x + z_{jk_2 ik_1}^x + z_{ik_1 jk_2}^y + z_{jk_2 ik_1}^y \geq 1 \quad \forall i, \forall j \neq i, \forall k_1 = 1, ..., m_i - 1, k_2 = 1, ..., m_j - 1 \tag{39}$$

$$x_{ik_1}^h - x_{ik_2}^l \leq \sum_{k=1}^{m_i-1} \sum_{r \in R_{ik}} A_{ir} w_{ir} \qquad \forall i, \forall k_1 \neq k_2 = 1, ..., m_i - 1 \tag{40}$$

$$y_{ik_1}^h - y_{ik_2}^l \leq \sum_{k=1}^{m_i-1} \sum_{r \in R_{ik}} B_{ir} w_{ir} \qquad \forall i, \forall k_1 \neq k_2 = 1, ..., m_i - 1 \tag{41}$$

$$\tilde{A}_i \geq \sum_{r \in R_{ik}} A_{ir} w_{ir} \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (42)$$

$$\tilde{B}_i \geq \sum_{r \in R_{ik}} B_{ir} w_{ir} \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (43)$$

$$LA_i \geq x_{ik}^h - \sum_{m \neq k} x_{im}^l \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (44)$$

$$LA_i \geq x_{ik}^h - x_{ik}^l \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (45)$$

$$LB_i \geq y_{ik}^h - \sum_{m \neq k} y_{im}^l \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (46)$$

$$LB_i \geq y_{ik}^h - y_{ik}^l \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (47)$$

$$LA_i \leq \tilde{A}_i + (1 - s_i)M \qquad \forall i, k_1 = 1, \dots, m_i - 1, k_2 = 1, \dots, m_i - 1, k_1 \neq k_2 \qquad (48)$$

$$LB_i \leq \tilde{B}_i + s_i M \qquad \forall i, k_1 = 1, \dots, m_i - 1, k_2 = 1, \dots, m_i - 1, k_1 \neq k_2 \qquad (49)$$

$$LA_i \leq (m_i a_i b_i AR)^{0.5} \qquad \forall i \qquad (50)$$

$$LB_i \leq (m_i a_i b_i AR)^{0.5} \qquad \forall i \qquad (51)$$

$$LA_i + LB_i \leq (2 m_i a_i b_i PF)^{0.5} \qquad \forall i \qquad (52)$$

$$w_{ir} = 0, 1 \qquad \forall i, \forall r \in R_{ik}, \forall k \qquad (53)$$

$$z_{ik_1 jk_2}^x, \; z_{ik_1 jk_2}^y = 0,1 \qquad \forall i, \forall j \neq i, \forall k_1 = 1, \dots, m_i - 1, k_2 = 1, \dots, m_j - 1 \qquad (54)$$

$$s_i = 0,1 \qquad \forall i \qquad (55)$$

$$x_i, y_i \geq 0 \qquad \forall i \qquad (56)$$

$$x_{ij}, y_{ij} \geq 0 \qquad \forall i, \forall j, i \neq j \qquad (57)$$

$$\bar{x}_{ik}, \bar{y}_{ik}, x_{ik}^h, x_{ik}^l, y_{ik}^h, x_{ik}^l \geq 0 \qquad \forall i, \forall k = 1, \dots, m_i - 1 \qquad (58)$$

The objective function (20) minimizes the transportation cost which is the product of the flow parameter and the rectilinear distance between the centroids of all pairs of departments. The distances between the centroids are obtained by constraints (21)-(24) while the centroids of the departments are calculated in constraints (25)-(26), based on the centorids of the chosen configurations. Those are obtained in constraints (27)-(28), while the centorids of the non-chosen configurations are set to zero in constraints (29)-(30). Constraints (31)-(32) set the dimension values of the two chosen configurations along both axes to the chosen configurations. Constraint (33) assures that exactly two configurations are chosen for each department, while constraint (34) ensures that the chosen configurations of department i contain in total exactly m_i machines. Constraints (35)-(39) prevent overlapping between all configurations. Constraints (40)-(41) assure that chosen configurations belonging to the same department are attached to each other. Constraints (42)-(43) set the value of the longest dimension of the chosen configurations of each department along each axis. The dimensions of each department are obtained in constraints (44)-(47). Constraints (48)-(49) enforce the L/T shape of the departments according to Definition 2, as they assure that at least one edge of a configuration belonging to a certain department will be attached along its entire length to the other configuration of that department. Constraints (50)-(52) are the regularity constraints. Finally, the binary and continuous decision variables are defined in constraints (53)-(58).

5   Conclusions

In this paper we proposed a new simultaneous approach for the facility layout problem (FLP). The contribution of the new approach is threefold. First, it considers simultaneously the location of the departments within the facility and the internal arrangement of the machines. Second, the non-linear area constraint of previous formulations is avoided and the new formulation has a linear structure. Third, the proposed approach allows non-rectangular department shapes, in particular L/T shaped departments. Two formulations are presented, for the rectangular and L/T shaped departments.

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